Capital Research

Imputation WACCs

Descriptions and Numerical Valuation Comparisons

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This short note describes the various imputation WACCs and then demonstrates how successful or otherwise they are in realistic valuation exercises. Once we see how robust or precious the various WACCs are with respect to their assumptions, we can get an insight into why some work quite well in practice whereas others are fraught with risk in their application.

1. Description of Imputation WACCS

The classical tax system only required we know two WACCs. These depended on whether the tax amount recognised in the cash flow was the actual tax after recognising the effect of tax shields ("geared tax") or whether the tax was based on no financing shields (an All Equity tax or an "ungeared tax"). The following diagram depicts this choice.

Figure 1: Classical WACCs

Tax recognition in Cash flow	Short desciption of WACC	WACC known as
Geared tax	Financial leverage WACC	Vanilla
Ungeared tax	All Equity finance WACC	Classical

The imputation tax system gives us further choice as the credits captured by shareholders are a before-personal tax amount so they have to be added back to the shareholders' cash flow in order to get an after company tax but before personal tax cash flow. The permutations are threefold: we can base the credits in the cash flow on the geared tax, the un-geared tax or ignore them altogether in the cash flow. In all cases the WACC corrects for any false assumption about the cash flow. Figure 2 describes this taxonomy. The actual formulae are in Table 1 below.

Figure 2: Imputation WACCs

Tax recognition in cash flow	Credit recognition in cash flow	Short desciption of the WACC	WACC known as	Abbreviation
Geared tax	Geared tax credit	Imputation as Vanilla WACC	Officer 3	Vanilla
	Ignored	Geared tax but credits ignored	-	Geared, Ignored
	Ungeared tax credit	Geared tax but All Equity credits	-	Geared, All Equity
Ungeared tax	Geared tax credit	Imputation as Classical WACC	Officer 4	Classical
	Ignored	All Equity tax but credits ignored	Officer 1	All Equity, Ignored
	Ungeared tax credit	Imputation as Effective Classical	Officer 2	Effective Classical

Table 1: WACC formulae

1. Classical Tax System

Geared cash flow		**Vanilla WACC**
Cash Flow	X_O ! $T(X_O - X_D)$	Tax shield in the cash flow
WACC	$\alpha R_E + (1-\alpha)R_D$	

Un-geared cash flow		**Classical WACC**
Cash Flow	X_O ! TX_O	All Equity (i.e. ungeared) cash flow
WACC	$(1-g)R_E + g(1-T)R_D$	

2. Imputation Tax System

Geared cash flow, geared tax credit		**Vanilla WACC**
Cash Flow	X_O ! $(1-\gamma)T(X_O - X_D)$	Tax shield in the cash flow & credits based on this geared tax
WACC	$(1-g)R_E + gR_D$	Officer 3

Geared cash flow, credits ignored in cash flow		**Geared but Ignored WACC**
Cash Flow	X_O ! $T(X_O - X_D)$	Tax shield in the cash flow but credits ignored in the cash flow
WACC	$(1-g)R_E \frac{(1-T)}{(1-T+\gamma T)} + gR_D$	-

Geared cash flow, un-geared tax credits		**Geared, All Equity WACC**
Cash Flow	X_O ! $T(X_O - X_D) + \gamma TX_O$	Tax shield in the cash flow but credits based on un-geared tax
WACC	$(1-g)R_E + g(1+\gamma T)R_D$	-

Un-geared cash flow, geared tax credit		**Classical WACC**
Cash Flow	X_O ! $TX_O + \gamma T(X_O - X_D)$	No tax shield in the cash flow but credits based on the actual geared tax
WACC	$(1-g)R_E + g(1-T)R_D$	Officer 4

Un-geared cash flow, credits ignored in cash flow		**All Equity, Ignored WACC**
Cash Flow	X_O ! TX_O	No tax shield in the cash flow and credits ignored in the cash flow
WACC	$(1-g)R_E \frac{(1-T)}{(1-T+\gamma T)} + g(1-T)R_D$	Officer 1

Un-geared cash flow, un-geared tax credits		**Effective Classical WACC**
Cash Flow	X_O ! $(1-\gamma)TX_O$	No tax shield in the cash flow and credits based on this un-geared tax
WACC	$(1-g)R_E + g(1-(1-\gamma)T)R_D$	Officer 2

As usual, the notation we use is

Notation	*Item*
Xo	EBIT(FCF version where capex for replenishment or the annualized equivalent thereof exactly offsets the depreciation deduction)
Xd	Interest payment on debt
Xo-Xd	Profit before tax
TXd	Interests Tax Shield (ITS)
Xo-X_G	Net operating profit after tax (nopat)
	Allocation
Xe	Cash flow to shareholders
Xd	Cash flow to debt holders
X_G	Cash flow to Tax Office as effective company tax
Xe+Xd	total excluding Tax (actual after tax cash flow to claimants)
	Rates
Re	Required return on equity
Rd	Required return on debt
Ra	Required return on the asset (the WACC)
g	Gearing (D/V)
	Valuation
E	Market value of Equity
D	Market value of Debt
V	Total Enterprise value (at market)

Observations:

From these formulae and Figures 1 and 2, some things become immediately obvious:

1. Only the Vanilla WACC (Officer 3) and the Effective Classical WACC (Officer2) have consistent assumptions between the interest tax shield in the cash flow and the quantum of the credits in the cash flow. All other combinations have inconsistent assumptions or ignore imputation altogether in the cash flow.

2. The Vanilla and the Effective Classical WACCs both closely resemble their classical tax counterparts by just substituting the effective tax rate after imputation $(1-\gamma)T$ for the classical tax, T. In the case of the Vanilla WACC, this occurs in the cash flow only and in the case of the Effective Classical it occurs in both the cash flow definition and the WACC formula.

3. All WACCs, except the Vanilla, include terms for correcting the mis-specified items in the cash flow. Only the Vanilla WACC accurately reflects all the tax effects in the cash flow, both classical tax and imputation credits, so only the Vanilla WACC has no tax terms for both the debt shield and the credits.

4. The Classical WACC is reproduced in the imputation set by accurately adding back the imputation credits but, as with the classical system, basing the cash flow on an un-geared tax. Because the credits are accurately reflected in the cash flow, there is no adjustment necessary in the WACC so the WACC remains as it is for the classical system.

5. There are imputation tax terms (gamma) in all WACCs except the Vanilla and Classical WACCs. This is because these are the only two WACCs that use the geared tax credit in the cash flow. Hence they do not need correcting terms in the WACC to compensate for erroneous assumptions in the cash flow.

6. The Vanilla WACC formula is based on a constantly geared perpetuity (as are all WACC formulae) but any error (or bias) between the Vanilla WACC formula and the valuation identity V = E+D will be due to the perpetuity formula assumption: either the infinite life span and/or the constant gearing assumption will be the source of any discrepancy.

Testing the Models

In order to test these models for robustness, a sample data set was constructed for a 20 year project. This sample data set has a constant set of revenue and operating costs in real terms. Volatility was then introduced into the revenue stream by randomly shocking the annual data by a random normal amount. The input rates data and the resulting WACCs (based on a target gearing of 29.4% - the actual gearing of the imputation base valuation) are as follows in Tables 2 and 3 respectively.

Table 2: Input rates data

MRP	6.00%
Risk Free	4.00%
Equity beta (levered)	0.75
Debt Premium	0.60%
Debt beta	0.10
Inflation	2.00%
Cost of Equity (levered, nom)	8.49%
Cost of Debt (levered, nom)	4.60%
Degeared asset beta	0.64
Degeared cost of capital	7.82%
Target Gearing (Debt/Total)	29.4%
Value of Imputation credits	50%
Gamma	0.50
Tax (statutory corporate rate)	30.%
Imputation adjusted Tax (effective tax rate)	15.0%

Table 3: WACC rates

Classical Tax System	Description	Rate	Known as:
Vanilla	geared interest tax shield in the cash flow	7.34%	
Classical	ungeared interest tax shield in the cash flow	6.94%	
Imputation Tax System			
Vanilla	geared interest tax shield and geared tax credits in the cash flow	7.34%	Officer 3
Geared but Ignored	geared interest tax shield in the cash flow but credits ignored in the cash flow	6.29%	
Geared, All Equity	geared interest tax shield in the cash flow but ungeared tax credits in the cash flow	7.55%	
Classical	ungeared interest tax shield in the cash flow but geared credits in the cash flow	6.94%	Officer 4
All Equity, Ignored	ungeared interest tax shield in the cash flow but credits ignored in the cash flow	5.88%	Officer 1
Effective Classical	ungeared interest tax shield in the cash flow and ungeared credits in the cash flow	7.14%	Officer 2

The regular cash flows used in the valuations are as follows: see Table 4.

Table 4: Regular Cash Flows

Operations

	0	1	2	…	19	20
	30/06/2004	30/06/2005	30/06/2006	…	30/06/2023	30/06/2024
Inflation index	1.000	1.020	1.040	…	1.457	1.486
capex	($750,000)					
revenue (real)		$275,000	$275,000	…	$275,000	$275,000
opex (real)		($150,000)	($150,000)	…	($150,000)	($150,000)

Prime Depreciation

Book Value Open		$750,000	$712,500	…	$75,000	$37,500
Depreciation		$37,500	$37,500	…	$37,500	$37,500
Book Value Close	$750,000	$712,500	$675,000	…	$37,500	$0

Residual Value

Depreciation		($37,500)	($37,500)	…	($37,500)	($37,500)
Closing Book Value						$0
Sale Price						$0
Profit/Loss on sale						$0
Tax on sale						$0
NCF from sale						$0

The resulting valuations are in Table 5. Note that the target gearing is set equal to the actual gearing of 29.4% in order to ensure that no error (at least in the imputation valuations) arises from actual gearing being different to target gearing, a common source of valuation discrepancies.

Table 5: Valuation Summaries

	Valuations	Gearing	Error
Classical			
D & E components	$1,225,498	32.6%	
Vanilla	$1,224,889	32.7%	-0.05%
Classical	$1,224,884	32.7%	-0.05%
Imputation			
D & E components	$1,362,299	29.4%	
Vanilla	$1,375,729	29.1%	+0.99%
Geared but Ignored	$1,337,001	29.9%	-1.86%
Geared, All Equity	$1,373,132	29.1%	+0.80%
Classical	$1,381,226	29.0%	+1.39%
All Equity, Ignored	$1,340,171	29.8%	-1.62%
Effective Classical	$1,378,426	29.0%	+1.18%

The base valuation, either classical or imputation, is taken as the valuation from the individual sum of debt and equity, i.e., the "D&E components" valuation. These are the present values of finite life series and are easily calculated. Even though the various WACC valuations are also valuations of finite life series, the derivation of each WACC (see Table 1) assumes perpetual cash flows. This is most easily seen in the derivation of the Vanilla WACC formula, viz:

Cash flow definition:

$$X_O - X_G = X_E + X_D$$

Return substitution (the perpetuity assumptions and Vanilla WACC definition):

$$\begin{aligned} X_E &= R_E \times E \\ X_D &= R_D \times D \\ X_O - X_G &= Wacc \times V \end{aligned}$$

Combining cash flow definition and returns:

$$Wacc \times V = R_E \times E + R_D \times D$$

Dividing through by V solves for the Vanilla WACC, namely

$$Wacc = R_E \times \frac{E}{V} + R_D \times \frac{D}{V}.$$

Hence there is a logical inconsistency when valuing a finite life stream with a WACC. However, in practice this can often be ignored because valuations of regular finite life streams often converge to a perpetuity value as the life span of the stream is increased.

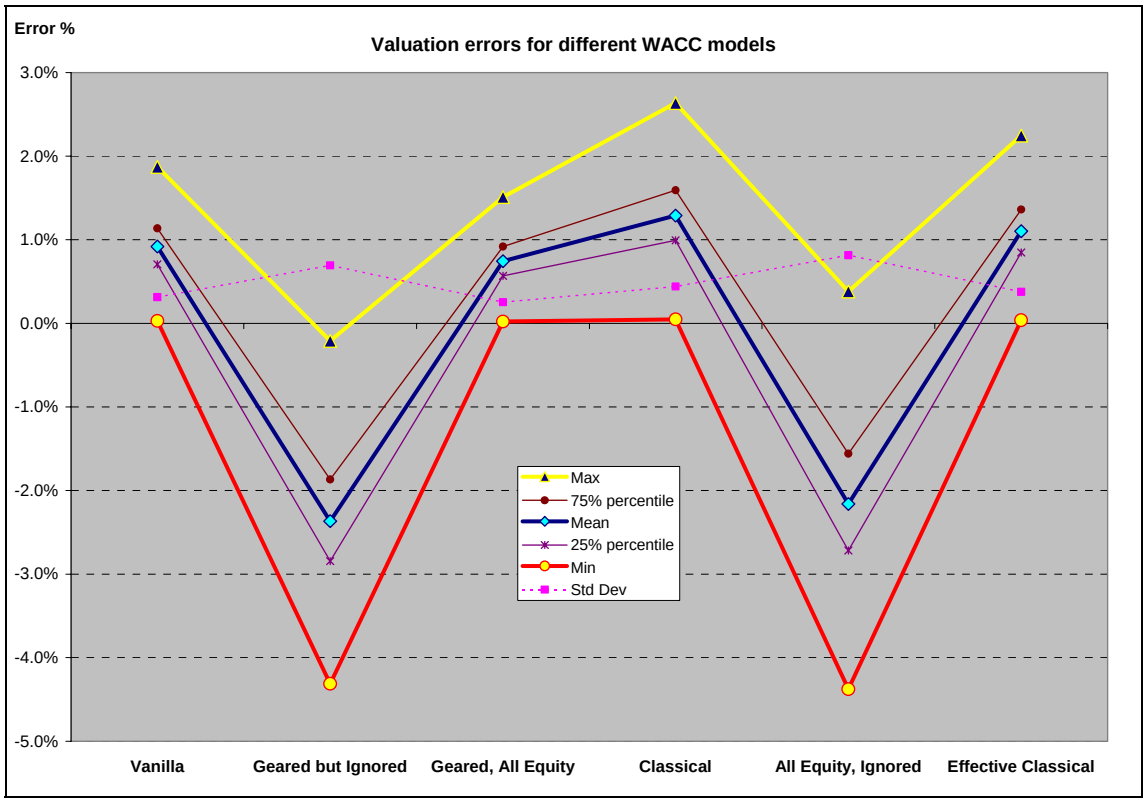
Another more insidious practical inconsistency is that the gearing is only the target gearing at time t=0. As the project moves through time, the gearing of the remaining future cash flow may change over time. It is possible to theoretically rebalance the capital structure through time in order to keep the gearing at a fixed target. It is our observation that practitioners rarely do this. In any event, our ambition here is to observe how a set of typical WACC valuations (which include these two assumptions of fixed gearing and perpetual life) compare to the strictly correct valuation of valuing the debt and equity as individual streams and aggregating the sum.

We will call the difference between the aggregate elements valuation of V = E+D and the Vanilla WACC valuation the "perpetuity bias" in the valuations. As all WACC valuations potentially have this bias to some extent, we will compare valuation errors after adjusting for this "bias". In the above valuation set (Table 5) the imputation WACC valuation bias = 0.99%.

Before reading too much into the comparison of these valuations as in Table 5, we shock the revenue stream (constant real value of $275,000) by substituting a normally distributed series of mean $275,000 and standard deviation of $50,000. This series was generated many times and the distribution of valuations was recorded (i.e. many copies of Table 5 were generated for different revenue streams).

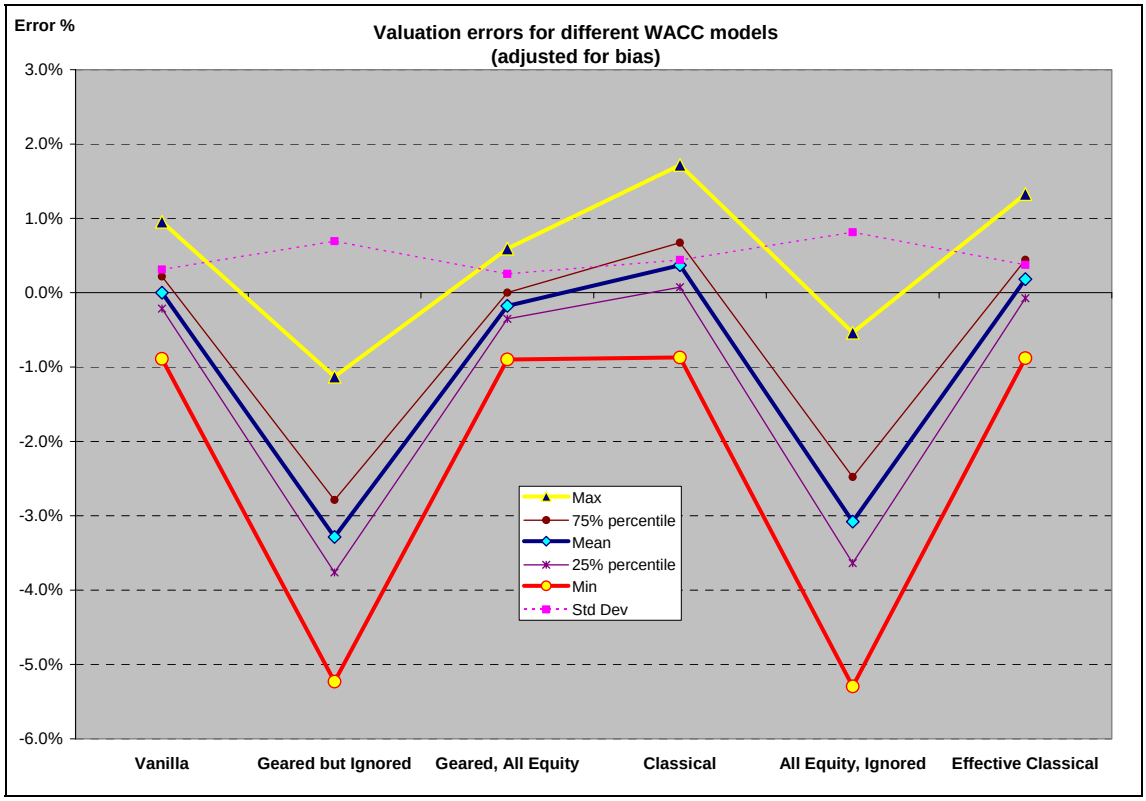
The summary results are plotted in Figure 3 for the raw results and Figure 4 for the bias adjusted results.

Figure 3: Raw Valuation Errors



As we expect, the mean bias is again approximately 1%. All errors were adjusted for this bias.

Figure 4: Bias Adjusted Valuation Errors



Observations on the Valuation models

1. Any model that ignores the credits in the cash flow and tries to correct for this in the WACC should be avoided at all costs. These models give substantially lower valuation estimates than all other models and higher spreads in errors. As the various WACC formulae are based on regular perpetuity models, these WACCs cannot cope with correcting for irregular credits that are ignored in the cash flow. The two "ignore" models (which include Officer 1) should both be ignored by practitioners. The other four models are deemed reasonably acceptable models by comparison.

2. The Vanilla, the Geared, All Equity, the Effective Classical and the Classical models all give similar results. The first two recognise financial gearing in the cash flow. The difference between them is the recognition of gearing in the credits. Vanilla recognises this gearing, Geared, All Equity does not. The Effective Classical approach ignores gearing in the cash flow and as such it is just an imputation version of the standard Classical WACC, adjusted for the effectively reduced corporate tax rate due to imputation credits. This model has the advantage of not having to introduce any financing into the cash flows. As such, this replicates the reason it was popular in the pre-imputation world.

3. The Classical model where credits are only recognised in the cash flow gives a wider spread of results among the four acceptable models. However, the difference is marginal and is likely to be well inside the tolerance for valuation error by practitioners.

The above is by no means a definitive comparison of the various models. It is rather an attempt to give a simple description of the various models and to explore their resulting robustness. If we had to recommend any one model it would be the Vanilla WACC model. It is the most consistent with respect to the input assumptions.

Any comments and suggestions for further comparisons are welcome.

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