

Valuation errors from nominal and real cash flows

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ERRORS ARISING FROM INCORRECT USE OF REAL & NOMINAL RATES

LOGIC

Inflation must be taken out of the nominal WACC *after* forming the nominal costs of capital and then applying the Fisher equation. Errors are made when forming a “real” WACC from real costs of capital. In addition, it is not uncommon to find people having inconsistencies in real and nominal values between their cash flows and rates. These errors seem more common in the perpetuity valuation of residual or “tail” cash flows that are often implicitly real. Cap rates in property valuations are a good example of an implicitly real valuation.

We demonstrate these issues and show that only the vanilla WACC has the ability to easily switch between real and nominal rates.

Real and Nominal WACCs

If we value individually the debt and equity components of a firm, then add these components to get the enterprise value, it should not matter at all whether we do the valuation in a real or nominal framework. As long as we use the Fisher equation, $(1+R_{nom}) = (1+R_{real})(1+i)$, for the translation between real and nominal rates, then the valuations will be identical. We then have an inflation factor in the cash flow (the numerator) that is perfectly matched by the same factor in the discount rate (the denominator) so that they perfectly cancel out. It does not matter then if we use real or nominal costs of equity and debt just so long as we have consistency with the cash flows.

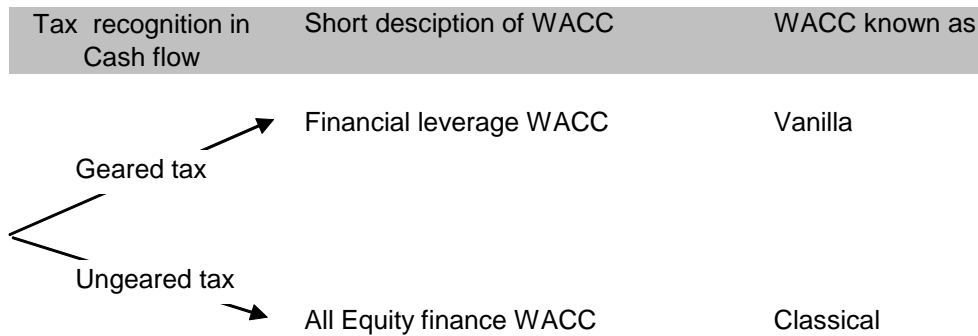
This is not always true when we form WACCs for an asset valuation. In that case we are performing a combined debt and equity valuation in the one calculation. It is most important then that we form a nominal WACC from nominal costs of capital and then take inflation out of the nominal WACC. We will make an error if we form a WACC from real costs of capital. This is *not* the same as a real WACC. The only exception is the Vanilla WACC which is robust in the order of taking out inflation. The Vanilla WACC stands out as the only one in which the real WACC is the same as the WACC formed from real costs of capital.

None of this discussion is changed under an imputation tax system. We can easily include imputation credits in the Vanilla WACC valuation by just adding back the (nominal) tax credits captured by shareholders. Equally, we can easily accommodate imputation tax credits in the Classical WACC valuation by using an effective tax rate after allowing for credit clawback instead of the full statutory rate.¹

The classical tax system only required we know two WACCs. These depended on whether the tax amount recognised in the cash flow was the actual tax after recognising the effect of tax shields (“geared tax”) or whether the tax was based on no financing shields (an All Equity tax or an “ungeared tax”). The following diagram depicts this choice.

¹ For further expansion on this, see our paper *Imputation WACCs – descriptions and numerical valuation comparisons*.

Figure 1: Classical WACCs



The imputation tax system has complicated this process but fundamentally we still face the same decisions: do we recognise finance in the cash flow and/or credits or do we ignore the effect of finance? The two valuation approaches are embodied in the following two formulae.

Table 1: Valuation approaches using WACC

Geared cash flow		Vanilla WACC
Cash Flow	$X_O - T(X_O - X_D)$	Tax shield in the cash flow
WACC	$(1 - g)R_E + gR_D$	

Un-geared cash flow		Classical WACC
Cash Flow	$X_O - TX_O$	All Equity (i.e. ungeared) cash flow
WACC	$(1 - g)R_E + g(1 - T)R_D$	

The imputation case can be included in this framework if we substitute the effective tax rate after imputation in place of the statutory tax rate.

As usual, the notation we use is

Notation	Item
X_o	EBIT(FCF version where capex for replenishment or the annualized equivalent thereof exactly offsets the depreciation deduction)
X_d	Interest payment on debt
$X_o - X_d$	Profit before tax
T	Statutory Company Tax Rate
$T.X_d$	Interest Tax Shield on debt (ITS)
$X_o - X_G$	Net operating profit after tax (nopat)
Capex	Capital expensed (and book value)
<i>Rates</i>	
R_e	Required return on equity
R_d	Required return on debt
R_a	Required return on the asset (the WACC)
g	Gearing (D/V)
<i>Valuation</i>	
E	Market value of Equity
D	Market value of Debt
V	Total Enterprise value (at market)

The issue being addressed here is the presence or absence of inflation in both the cash flow and WACC data sets. The ungeared or all-equity version of the valuation formula using *nominal* data is

$$NPV = -\text{capex} + \sum \frac{X_{o,t}(1-T)}{((1-g)R_e + g(1-T)R_d)^t}$$

When we perform a valuation using the geared or Vanilla WACC with the ITS in the cash flow, the formal valuation formula is

$$NPV = -\text{capex} + \sum \frac{X_{o,t} - TX_{o,t} + TX_{d,t}}{((1-g)R_e + gR_d)^t}$$

and it does not matter in this case whether the data is all nominal or all real. To see why this is so, consider taking inflation out of the nominal Vanilla WACC, as follows.

$$\begin{aligned} WACC^{\text{Nom}} &= (1-g)R_E^{\text{Nom}} + gR_D^{\text{Nom}} \\ 1 + WACC^{\text{Nom}} &= (1-g)(1 + R_E^{\text{Nom}}) + g(1 + R_D^{\text{Nom}}) \\ &= (1-g)(1 + R_E^{\text{Real}})(1+i) + g(1 + R_D^{\text{Real}})(1+i) \\ &= [(1-g)(1 + R_E^{\text{Real}}) + g(1 + R_D^{\text{Real}})](1+i) \\ &= [1 + (1-g)R_E^{\text{Real}} + gR_D^{\text{Real}}](1+i) \\ &= [1 + WACC^{\text{Real}}](1+i). \end{aligned}$$

We see that the Vanilla WACC formed with the real costs of debt and equity is identical to the one we get by applying the Fisher equation to the nominal WACC. This would still hold if the capital structure of the enterprise was quite involved, eg, if it had issued various forms of equity and various debt maturities (as indeed do many enterprises). In that case the Vanilla WACC would be a weighted sum of numerous costs of capital, namely

$$WACC = \sum g_i R_i \quad \sum g_i = 1$$

but we would still have the important property that

$$1 + WACC = 1 + \sum g_i R_i = \sum g_i (1 + R_i).$$

We need this property in order to apply the Fisher equation, namely

$$(1 + WACC)(1+i) = \sum g_i (1 + R_i)(1+i).$$

The reason this works is the absence of any tax terms in the Vanilla WACC. If there are any tax terms at all in the WACC then the Fisher equation cannot be applied to the components of the WACC. Instead, we must use the Fisher equation on the whole WACC result, as

$$(1 + WACC^{\text{Nom}}) = (1 + WACC^{\text{Real}})(1+i).$$

In that case, all valuations will be consistent regardless of the form of the WACC as then the valuation will be

$$NPV = \sum \frac{\text{CashFlow}_t^{\text{Nom}}}{(1 + \text{WACC}^{\text{Nom}})^t} = \sum \frac{\text{CashFlow}_t^{\text{Real}} (1 + i)^t}{(1 + \text{WACC}^{\text{Real}})^t (1 + i)^t} = \sum \frac{\text{CashFlow}_t^{\text{Real}}}{(1 + \text{WACC}^{\text{Real}})^t}$$

Common Slip Ups

The most common place we see an error between real and nominal valuations is in the tail component of a valuation. It is quite common to see a valuation done over 10 years of nominal cash flows and the tail value representing the capitalised value of years 11+. As it is not uncommon to see over 50% of a valuation contributed by the tail value, we ought to at least get that part of the valuation correct.

Typically cash flow #11 is an inflated amount over cash flow #10 and it is assumed to grow at a given rate of inflation thereafter. The valuation we most often see is of the form

$$NPV = \sum_{t=1}^{10} \frac{\text{CashFlow}_t}{(1 + \text{WACC})^t} + \frac{1}{(1 + \text{WACC})^{10}} \text{pv}_{10}(\text{TAIL})$$

$$\text{pv}_{10}(\text{Tail}) = \left(\frac{\text{CashFlow}_{11}}{\text{WACC}} \right).$$

If the cash flows are nominal which is often the case, then this is an error (Error #1). The reason is because the tail term is valued as a perpetuity and this is implicitly real. Note that the present value of the tail is a capitalised amount at year 10 so it is in dollars of the day at year 10.

To see why the error occurs, recall that the present value at time zero of a constant and regular stream, C, starting at period 1, is

$$\text{pv} = \frac{C}{r}$$

and the present value at time zero of a regular growth stream, where the next (i.e. period 1) cash flow is C with a constant periodic growth rate thereafter of g, is

$$\text{pv} = \frac{C}{r - g}.$$

If cash flows #11+ are due to grow at the rate of inflation then the capitalised value using the nominal WACC should be as follows (CORRECT valuation) where the periodic growth rate is just inflation, i

$$NPV = \sum_{t=1}^{10} \frac{\text{CashFlow}_t}{(1 + \text{WACC}^{\text{Nom}})^t} + \frac{1}{(1 + \text{WACC}^{\text{Nom}})^{10}} \left(\frac{\text{CashFlow}_{11}}{\text{WACC}^{\text{Nom}} - i} \right)$$

$$\text{pv}_{10}(\text{TAIL}) = \frac{\text{CashFlow}_{11}}{\text{WACC}^{\text{Nom}} - i}.$$

Notice that it is also an error to value the tail by capitalising a constant real cash flow #11 at a real WACC (Error #2), i.e. it is an error to value the tail as

$$pv_{10}(\text{TAIL}) = \frac{\text{CashFlow}_{11}}{\text{WACC}^{\text{Real}}}.$$

We know this is an error because we know that typically real rates are not just the difference between nominal rates and inflation. There is also an interaction term to contend with, namely

$$\begin{aligned} (1 + \text{WACC}^{\text{Nom}}) &= (1 + \text{WACC}^{\text{Real}})(1 + i) \\ \text{WACC}^{\text{Nom}} &= \text{WACC}^{\text{Real}} + i + \text{WACC}^{\text{Real}} \times i. \end{aligned}$$

From this Fisher equation, we can see why the real rate version of the tail is in error. We can rearrange this Fisher equation result as follows:

$$\begin{aligned} \text{WACC}^{\text{Nom}} - i &= \text{WACC}^{\text{Real}} + \text{WACC}^{\text{Real}} \times i \\ &= \text{WACC}^{\text{Real}} (1 + i). \end{aligned}$$

Using this last result, the correct valuation of the tail is then

$$\begin{aligned} pv_{10}(\text{TAIL}) &= \frac{\text{CahFlow}_{11}}{\text{WACC}^{\text{Nom}} - i} \\ &= \frac{\text{CashFlow}_{11}}{\text{WACC}^{\text{Real}} (1 + i)}. \end{aligned}$$

Hence, valuing the tail by capitalising the perpetual real cash flow #11 at the real rate of $\text{WACC}^{\text{Real}}$ is in error by one inflation factor. This indicates the subtle nature of the error. A real WACC valuation done on cash flow #11 is in dollars of the day at period 11, not at period 10. However, the valuation is located at period 10 so it has to be deflated by one period of inflation and this is captured in the $(1+i)$ term in the correct valuation.

Note that all of the above discussion applies to real estate capitalisation. Dividing the rent by the capitalisation rate to value a property is implicitly assuming no growth (or more accurately, embedding the growth rate in a net capitalisation factor). This includes not explicitly including the CPI forecast. Hence, the cap rate for property is implicitly of the form of a real perpetuity valuation.

A numerical example follows.

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Numerical Examples

This example is based on case in which the analysts for the transaction made a major error in valuing an implicit real tail at a nominal WACC within a high inflation environment (1975-1985). This substantially understated the asset value².

The input rate data are presented in Table 2a and the corresponding set of WACC rates are presented in Table 2b. We do the valuation as an ungeared one under imputation. The associated Classical WACC is adjusted for the effective tax rate after imputation of 19.5% and we call this the Effective Classical WACC. Note that the inflation rate is set at 5%. This will exaggerate the errors under current inflation rates of 2-3% but gives a good indication of the actual valuation errors made around the period 1980+.

Table2a: Cost of Capital input data

MRP	6.00%
Risk Free	8.00%
Equity beta (levered)	0.75
Debt Premium	0.60%
Target Gearing (Debt/Total)	25.0%
Inflation	5.0%
Gamma	0.35
Tax	30%
Debt beta	0.10
Cost of Equity (levered, nom)	12.5%
Cost of Debt (levered, nom)	8.6%
Imputation adjusted Tax	19.5%

Table 2b: WACC Rates

Classical Tax System		Nominal	Real
Vanilla	WACC (geared interest tax shield in cash flow)	11.51%	6.20%
Classical	WACC (ungeared interest tax shield in cash flow)	10.87%	5.59%

Imputation Tax System

Vanilla	WACC (geared interest tax shield and geared tax credits in cash flow)	11.51%	6.20%	Off. 3
Geared but Ignored	WACC (geared tax shield in cash flow but credits ignored in the cash flow)	10.29%	5.04%	
Geared All Equity	WACC (geared tax shield in cash flow but ungeared tax credits in cash flow)	11.74%	6.42%	
Classical	WACC (ungeared tax shield but geared credits in cash flow)	10.87%	5.59%	Off. 4
All Equity Ignored	WACC (ungeared tax shield but credits ignored in the cash flow)	9.65%	4.43%	Off. 1
Effective Classical	WACC (ungeared tax shield and ungeared credits in cash flow)	11.09%	5.80%	Off. 2

The Fisher equation of $(1+WACC^{Nom}) = (1+WACC^{Real})(1+i)$ is used to calculate all the real WACC data.

The cash flows for the project are presented in Table 3. All cash flows are assumed to be 30 June of the corresponding period. The valuation date is 30 June 2004. The capex payment is a single

² The author was an expert for a taxation dispute based on an asset sale in which the valuation was wrong. The spreadsheet supplied to support the case demonstrated that the client bought the asset based on an erroneous tail valuation. Notwithstanding their dispute with the tax office, they accidentally “won” by buying the asset at a “bargain” price due to the error – see Error#1 below.

payment at 30 June 2004. It is written off over 10 years on a prime basis. The perpetuity for the tail period (years 11+) cannot have a lumpy capex investment every 10 years so an annual equivalent or maintenance capex value for the lumpy capex was calculated and used in the perpetuity.

Table 3: Cash flow series

DATA SERIES (\$'000)												
	0	1	2	3	4	5	6	7	8	9	10	11 ...
	30/06/04	30/06/05	30/06/06	30/06/07	30/06/08	30/06/09	30/06/10	30/06/11	30/06/12	30/06/13	30/06/14	30/06/15 ...
Inflation	1.00	1.05	1.10	1.16	1.22	1.28	1.34	1.41	1.48	1.55	1.63	1.71 ...
Capex	-1,000											
Capital maintenance (AEV of renewal capex)												-127.3 ...
revenue(real)		275	300	310	320	330	340	350	350	375	400	400 ...
opex(real)		-150	-150	-150	-150	-150	-150	-150	-150	-150	-150	-150 ...

PRIME DEPRECIATION (\$'000)												
<i>Capex</i>												
Book Value Open		1,000	900	800	700	600	500	400	300	200	100	\$0 ...
Depreciation(Prime)		100	100	100	100	100	100	100	100	100	100	\$0 ...
Book Value	1,000	900	800	700	600	500	400	300	200	100	0	\$0 ...
Close												
Prime Total		100	100	100	100	100	100	100	100	100	100	\$0 ...

UN-GEARED FIRM, EFFECTIVE CLASSICAL TAX (\$'000)												
Revenue (nom)		288.75	330.75	358.86	388.96	421.17	455.63	492.49	517.11	581.75	651.56	684.14 ...
Opex (nom)		-157.5	-165.38	-173.64	-182.33	-191.44	-201.01	-211.07	-221.62	-232.70	-244.33	-256.55 ...
Depreciation (nom)		-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	...
Maintenance capex												-217.60 ...
Taxable income (nom)		31.25	65.38	85.22	106.64	129.73	154.62	181.42	195.49	249.05	307.22	209.98 ...
Tax effective (nom)		-6.09	-12.75	-16.62	-20.79	-25.30	-30.15	-35.38	-38.12	-48.56	-59.91	-40.95 ...
Capex	-1,000										0	...
Cash flow after tax		125.16	152.63	168.60	185.84	204.43	224.47	246.04	257.37	300.48	347.32	386.64 ...

With these data, we perform the valuation based first on the nominal Effective Classical WACC of 11.09%. About 2/3 of the asset value comes from the tail. We had better get it right.

Table 4: Valuation Results

		Error
Capex	(\$1,000,000)	
NPV (years 1-10)	\$1,188,050	
CORRECT VALUATION		
Pv(Tail @ year 10)	\$6,343,586	
Pv(Tail @ year 0)	\$2,215,091	
NPV	\$2,403,142	
ERROR #1 - IGNORING INFLATION AS GROWTH		
Pv(Tail @ year 10)	\$3,484,814	
Pv(Tail @ year 0)	\$1,216,848	(\$998,243)
NPV	\$1,404,899	
ERROR #2 - VALUING YEAR 11 AS REAL		
Pv(Tail @ year 10)	\$6,660,765	
Pv(Tail @ year 0)	\$2,325,846	\$110,755
NPV	\$2,513,896	

Error #1 is very large – capitalising a nominal tail perpetuity at a nominal cost of capital appears sensible until we realize that we have not allowed for the nominal growth in the cash flows. This error under-values the asset by almost \$1 million, a 56% under-valuation. This is, unfortunately, an

all too common error. It has the superficial feel of being consistent by valuing nominal cash at a nominal rate which is probably the enticement into error.

If we then assume that there is no growth in the real tail cash flow and capitalise that real data at the real WACC equivalent of 5.80%, we over-value the asset by \$110,755 i.e. we commit Error #2. Once again this feels consistent because we have valued real cash at a real rate. The error is one period of inflation in excess of the actual tail value – in this example 5% of \$2,215,091. This is the inflation component arising from valuing the tail in real terms starting at the next cash flow, i.e. cash flow #11, but not allowing for the inflation across the first period of the perpetuity, time $t=10$ to time $t=11$ in this example.

Summary

It is very easy to make valuation errors when using real and nominal data. A common error is to build real WACCs from real costs of debt and equity. This is a mistake in all cases except the Vanilla WACC. This is the only WACC which gives the correct real value by building it out of real inputs. We have ignored the blatant error of mistakenly using rates and cash flows that are inconsistent with respect to inflation. That happens but often it is also subtle. For example, the valuation of real estate by the capitalisation method is a blend of nominal data (the contracted next rental payment) and implicitly real discount rates as in the cap rate. We value the property today based on the next period rent so that the implied cap rate is actually a real rate accumulated by one period of inflation – as in the correct tail valuation formula above.

The major errors of this type we see are in the tail components of valuations. These are perpetuity values based on growth assumptions which typically include inflation. The most significant error is to not allow for inflationary growth in the nominal valuation. Just using a consistent blend of real or nominal rates and cash flow does not correct the error. Using real cash and real rates substantially reduces the error because the missing growth from inflation is then included but it also introduces a timing miss-match which has to be recognised.

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