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Valuing Debt Interest Tax Shields and De-gearing Formulae

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DEBT INTEREST TAX SHIELDS INCORRECTLY VALUED

LOGIC

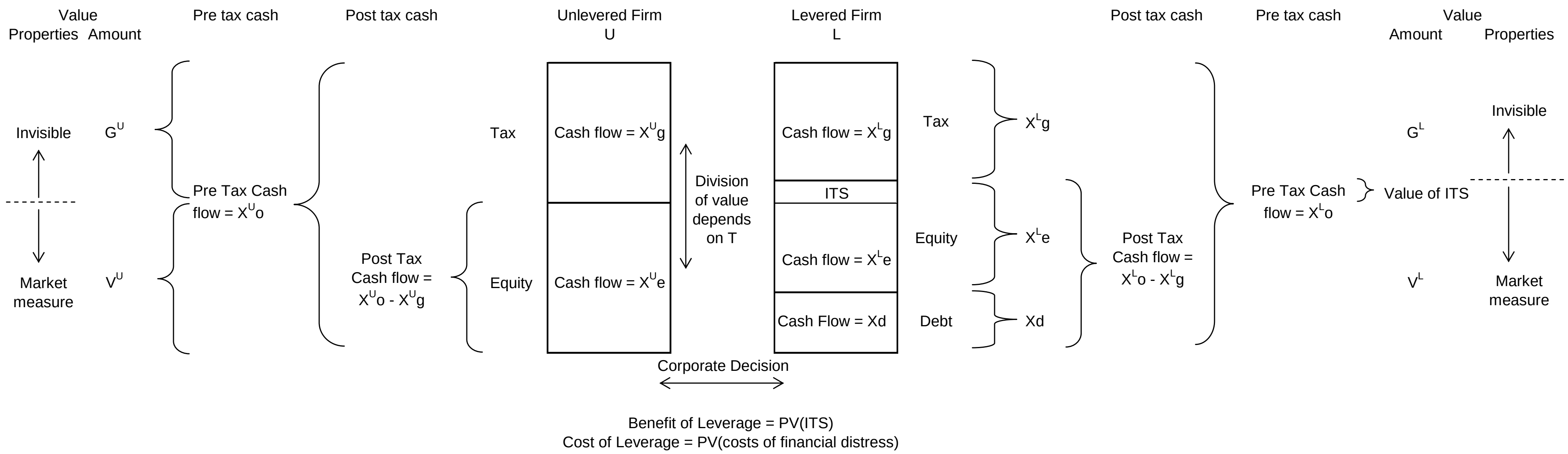
Debt interest tax shields (ITS) should be valued using the difference between two values in which the cash flows are valued at the cost of equity. We will demonstrate which two streams and which costs of equity to choose. In addition, the standard beta de-gearing formula makes the heroic assumption of debt shields are net value add to shareholders. The ITS is otherwise taxation dollars in the hands of the private sector. In the special case of a perpetuity the value of the ITS reduces to the cost of debt as found in standard text books. The tax shield on debt is a cash flow to *shareholders,* not debt holders. When costs of equity are taken from the market place, as is usually the case, the cash flow (dividends and credits) and prices being used to calculate capital gains and losses includes a component for the tax shield on debt. If one valued the equity only of a business, the tax shield on debt would be a component of the cash flow to the shareholders. This is always the case. But, the appropriate cost of capital is as ever an attribute of the asset (in this case a debt-derived cash flow) and not of who owns the asset (ie shareholders).

As usual, the notation we use is

Notation	*Item*
Xo	EBIT(FCF version where capex for replenishment or the annualized equivalent thereof exactly offsets the depreciation deduction)
Xd	Interest payment on debt
Xo-Xd	Profit before tax
TXd	Interests Tax Shield (ITS)
Xo-Xg	Net operating profit after tax (nopat)
	Allocation
Xe	Cash flow to shareholders
Xd	Cash flow to debt holders
Xg	Cash flow to Tax Office as effective company tax
Xe+Xd	total excluding Tax (actual after tax cash flow to claimants)
	Rates
Re	Requited return on equity
Rd	Required return on debt
Ra	Required return on the asset (the WACC)
g	Gearing (D/V)
	Valuation
E	Market value of Equity
D	Market value of Debt
V	Total Enterprise value (at market)
G	Market's value of government "pseudo equity" as a stream of company tax payments
	Leverage
U	Un-levered (all-equity) enterprise
L	Levered enterprise (capital structure includes debt)

The following diagram describes the logical set up. Everything is standard except for the inclusion of the (unseeable) capital value of the government's company tax collection. We can see the annual flows but not the capital value of those flows. Its value is the present value *as if* the tax collections were in the possession of the private sector. It is not necessarily the value the government itself would place on the tax collections, even if it ever entertained the notion of valuing the tax stream.

Logical Set Up



The ITS is clearly a component of the cash flow to *shareholders* of a levered firm. Companies can deliver this ITS to shareholders at the expense of government. However, it is not a free good as it comes with a cost of leverage, often called a cost of distress.

We assume that the pre-tax operating income, Xo, is independent of the leverage of the firm so then $X^{U}o = X^{L}o = Xo$.

For all taxable companies, the fortunes of the shareholders are directly correlated with the fortunes of the tax office as a collector of company tax. Only equity pays company tax. A good year for the shareholders is a good year for the tax office, and vice versa. The two claimants currently share the profits superficially as 70-30 but in the past it has been different, such as a 61-39 share. We say "superficially" because the introduction of imputation credits has meant that some company tax payments have been just withholdings of personal tax. We estimate elsewhere (The Value of Imputation Tax Credits – Update 2004) that the effective tax rate is now 19% instead of the statutory rate of 30%. In the logic below, the tax rate T is meant to be the *effective* rate, not the statutory rate. The statutory rate is used to calculate the ITS and credits are added back to the cash flow going to shareholders. We put explicit imputation tax considerations aside for the moment.

Each dollar paid by shareholders to the government as company tax has the opportunity cost of equity. If shareholders could retain this capital it would earn the shareholders' cost of equity. Equally, with a reduction in the company tax rate, each dollar foregone from government as company tax to the benefit of shareholders has the prevailing equity risk. Hence, the market value for the company tax payments is just the tax payments capitalised at the appropriate opportunity cost of equity. We call this market value the government's "pseudo equity" value for its tax collections and denote it by G. The underlying assumption is that the tax payments in the hands of the shareholders would not change the risk properties of the expanded equity. Australia is such a small and open economy in world terms that even if Australian company tax was totally abolished, it is very unlikely that this move would alter the risk-return trade-off for equity.

The effect of leverage on enterprise value is expressed as the net gain over the un-levered value,

$$V^L = V^U + pv(ITS) - cost(Leverage).$$

With the two assumptions that:

1. the cost of leverage is zero
2. leverage does not alter pre-tax cash flow

we must have preservation of total value,

$$V^U + G^U = V^L + G^L \quad \text{or} \quad V^L = V^U + G^U - G^L.$$

So $pv(ITS) = G^U - G^L$ which is no more than saying that the benefit of the ITS is the value of otherwise government tax collections in the possession of the shareholders. Governments can voluntarily forego company tax collections by reducing the company tax rate. In doing so more value is captured in the enterprise by the shareholders at the expense of government. However, there may be offsetting tax collections due to higher economic activity on the part of legal entities forming as companies and some lower company tax collections are offset by higher personal tax collections under our imputation tax system. There also may be commensurate rebalancing of the cost of debt if equity becomes more attractive to investors.

These are general equilibrium problems which are much too hard to address here. In any event, we only need a partial equilibrium solution:

> *given* the prevailing market costs of debt and equity under the current regime, how do we value the ITS?

The opportunity cost valuations for tax collections from an un-levered and a levered equivalent company are just

$$G^U = pv(X^U_g, R^U_e) \quad \text{and} \quad G^L = pv(X^L_g, R^L_e)$$

So the value of the ITS is just the difference between two equity values discounted at the appropriate cost of equity applicable to the geared or un-geared business,

$$Pv(ITS) = pv(X^U_g, R^U_e) - pv(X^L_g, R^L_e). \quad (1)$$

This formula applies to *any* stream of cash flows, not just regular annuities or perpetuities. In the case of perpetuities however, we can reduce this to a simple formula (as found in many text books).

For a perpetuity in which Xg = T(Xo – Xd) and admitting the case of Xd = 0 for an un-levered firm, we get

$$G = \frac{T(Xo - Xd)}{\text{Re}} = \frac{T(Xo - Xd)(1-T)}{(1-T)\,\text{Re}} = \left(\frac{T}{1-T}\right)E$$

and the government pseudo value of its company tax stream is just the tax portion of the "grossed-up" equity value E/(1-T).

As an example, the aggregate market value of the ASX stock market on 8 Dec 2004 was $922 billion so at an effective company tax rate of 19% this equates to a grossed up equity value of $1,138 billion and an equivalent government pseudo equity value of $216 billion.

Because the un-levered firm value is the same as the equity value, we can value the ITS as

$$pv(ITS) = \left(\frac{T}{1-T}\right)(V^U - E^L)$$

But from the whole enterprise view (recalling the assumption of no costs of leverage) we have

$$V^L = V^U + pv(ITS) = V^U + \left(\frac{T}{1-T}\right)(V^U - E^L)$$

and re-arranging the algebra we have

$$E^L + D^L = V^U + TD^L$$

so we have the text book result that pv(ITS) = TD^L.

Because the ITS per annum is just TXd = $TRdD^L$ we must conclude that the appropriate discount rate for the ITS is Rd, because

$$pv(ITS) = TD^L = \frac{TRdD^L}{Rd} = \frac{TXd}{Rd}.$$

The textbook results are correct but for the wrong reasoning. The risk of the ITS is generally not the risk of debt interest. We only have tax to shield when a profit is made but the interest must be paid on the debt regardless. Hence the ITS has equity risk, not debt risk. However, in a perpetuity we have to assume that we always make profits (perpetual loses are not worth contemplating) so we assume away the equity risk of the ITS and if we never make loses, the risk of the ITS reduces to the risk of debt. Generally we have to value the ITS as the difference between the two government tax values, as in equation (1) using the cost of equity.

Finally, we can get the correct de-gearing formula between levered and un-levered equity via the above analysis.

From the valuation identity we have

$$\mathrm{E}^{\mathrm{L}} + \mathrm{D}^{\mathrm{L}} = \mathrm{V}^{\mathrm{U}} + \mathrm{TD}^{\mathrm{L}}$$
$$\mathrm{V}^{\mathrm{U}} = \mathrm{E}^{\mathrm{L}} + \mathrm{D}^{\mathrm{L}} - \mathrm{TD}^{\mathrm{L}}.$$

From the definitions of perpetuity returns and WACC, we get

$$Xe^{L} = \mathrm{Re}^{L} E^{L}$$
$$Xd^{L} = Rd^{L} D^{L}$$
$$Xo(1-T) = RaV^{U}$$

From the total cash flow and tax calculation identity we get

$$Xo = Xe^{L} + Xd^{L} + T(Xo - Xd^{L})$$

and after combining the cash flow and returns expressions we get the solution

$$\mathrm{Re}^{L} = Ra + (Ra - Rd^{L})(\frac{D^{L}}{E^{L}})(1-T)\,. \tag{2}$$

If we also invoke the CAPM then this re-gearing can be expressed in terms of asset and debt betas as

$$\beta e^{L} = \beta a + (\beta a - \beta d^{L})(\frac{D^{L}}{E^{L}})(1-T) \tag{3}$$

This is the correct de-gearing formula *if* we accept the assumptions, particularly that there is a costless gain to leverage.

These assumptions are hard to accept at face value. Increased leverage will typically come at the cost of increased financial risk. We might argue a practical solution in that over a reasonable range of gearing levels, there is unlikely to be any visible cost of leverage. At the non-geared (all equity) level there is the opportunity cost of a lazy balance sheet or financial slack generally. However, we must also recognise that financial slack may be a benefit in that it gives companies freedom of quick action on investment prospects. Well-managed companies will strike an acceptable balance between these two competing forces. In contrast, at the high geared end, there will be significant costs from the high risk of bankruptcy and associated agency costs. Different types of assets will support different levels of financial risk. Within this reasonable range of gearing, we might observe that there is a net benefit from the ITS but at the extremes of gearing the cost of leverage out-weigh the benefits of leverage.

This is the *trade-off theory* of financial leverage. It seems to work well in explaining differences in gearing across industries. In contrast, it does not explain why the better performing companies within an industry tend to have lower gearing than others within that industry. The *pecking order theory* of leverage better explains this observation. Here companies prefer internal finance and manage their dividend payout policy along with their short term debt policy in order to manage their call on external finance in an optimal manner.

A possible reconciliation of the two theories is that the trade-off theory explains the base or average level of gearing based on the investment attributes of the *asset* and the pecking order theory explains the skill of successful (or otherwise) *executives* at managing the cash flow within companies. There will be investor expectation to be met for things like payout ratios so we expect these to be reasonably similar within industries. The less skilful managers will want to keep up with their peers and in the process have to rely on more external debt, increasing their relative gearing.

If there is an optimal gearing level it is likely to be a very weak optimum (the market value "hill" is very flat when plotted against gearing levels as in Figure 1a) so approximately the Modigliani and Miller (M&M) propositions apply over a range of gearing levels. The increased risk of leverage more or less cancels out the apparent gains via the ITS leaving market value approximately unchanged with changing gearing levels, at least when we stay away from extreme levels of gearing. The following two diagrams describe these competing cases.

Figure 1a There is no net gain from the ITS

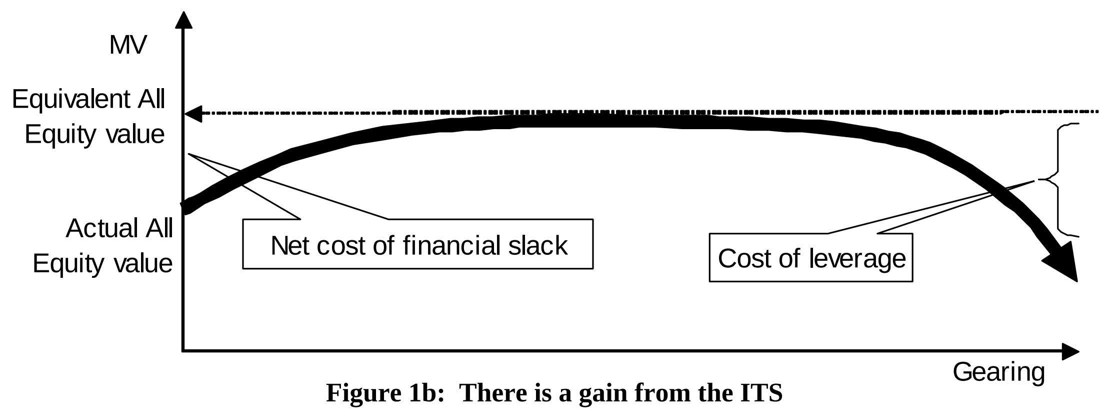
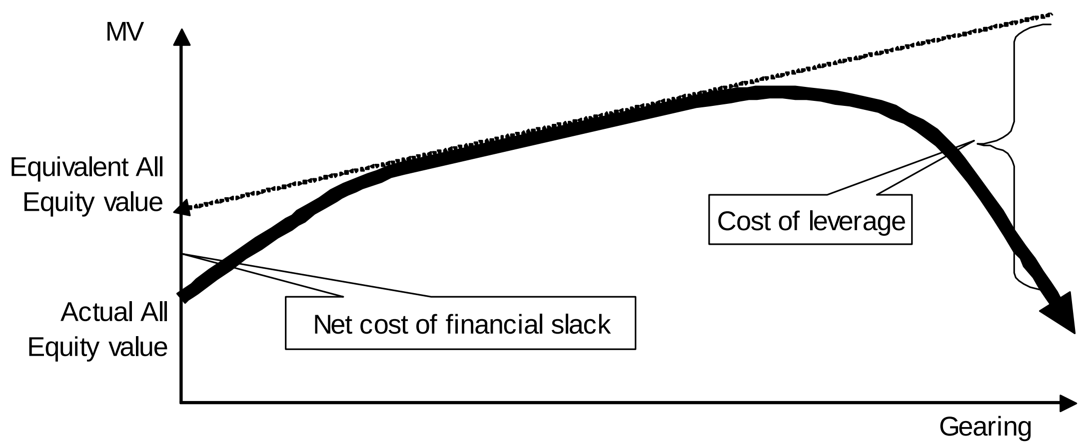


Figure 1b: There is a gain from the ITS



When we use the standard de-gearing formula, either equation (2) or (3) then we are calculating the equivalent all-equity value point by extrapolating the line in Figure 1b back to the zero gearing point. We re-gear from this point back to another chosen gearing level (typically done when comparing two companies with different gearing levels) which will again be a point along this straight line. We are assuming in this case that

1. operating income is independent of leverage, and
2. there is a net gain from the ITS, $V^L = V^U + pv(ITS)$

An alternative approach is to estimate the after tax WACC using an existing gearing level and then using that estimate as the WACC for a different gearing level. We would be assuming two things in this case:

1. operating income is independent of leverage, and
2. there is no net gain from the ITS, $V^L = V^U$

We would be then assuming the following:

$$V^L = \frac{Xo(1-T)}{WACC^L} = \frac{Xo(1-T)}{Ra} = V^U$$

We do not get a simple, clean formula for the de-gearing process in this case. Instead we have a numerical approach of equating WACCs and extracting asset betas from the solution, viz

$$\begin{aligned} Ra &= \alpha \mathrm{Re}^L + (1-\alpha)(1-T)Rd^L \\ \alpha &= E^L / V \\ \mathrm{Re}^L &= Rfree + \beta e^L MRP \\ Rd^L &= Rfree + \beta d^L MRP \\ Ra &= Rfree + \beta a MRP \end{aligned} \tag{4}$$

In summary, the value of the ITS is simply the difference between two otherwise flows to government in the hands of the private sector. Whether or not the ITS is a net gain to a company depends on the assumption of whether or not it comes at a cost and how much that cost offsets the value of the ITS.

These assumptions are embodied in the gearing and de-gearing formulae commonly used in WACC estimates. The classic de-gearing formula assumes there is no cost to the ITS. Whether or not this is an assumption of substance depends on how far the gearing is shifted between the "before" and "after" case. Not much weight should be given to the intermediate step of the all equity WACC calculation because it is just an *equivalent* all equity WACC and may differ substantially from the WACC of an actual all equity enterprise.

Numerical examples follow for both perpetuity and non-perpetuity cases.

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Numerical Examples

a. Perpetuity

Notation	*Item*	**U**	**L**	*Comment*
Xo	ebit (FCF)	$1,000.00	$1,000.00	
Xd	interest	$0.00	$138.00	
Xo-Xd	pbt	$1,000.00	$862.00	
Xg	tax (@ 30%)	$300.00	$258.60	
TXd	ITS		$41.40	
Xo-Xg	nopat	$700.00	$741.40	A perpetual positive profit
	Allocation			
Xe	Share holders	$700.00	$603.40	
Xd	Debt holders	$0.00	$138.00	
Xg	Tax Office	$300.00	$258.60	
Xe+Xd	total ex-tax	$700.00	$741.40	
	Valuation			
E	Equity	$9,211	$7,111	
D	Debt	$0	$3,000	
V	Total Market	$9,211	$10,111	ITS comes at zero cost (assumption)
	Govt cash flow	**$300.00**	**$258.60**	
	Discount rate	**7.600%**	**8.485%**	
G	**Govt "equity"**	**$3,947**	**$3,048**	
	pv(ITS)		**$900**	Difference between two equity values
	V+G+pv(ITS)	**$13,158**	**$13,159**	Slight rounding error – total value is preserved (nb: no cost of leverage)
	ITS			
	Valuation "Gain"		**$900**	ITS cap rate is the same as the cost of debt in a perpetuity
	Cap rate of gain		**4.6%**	
	Market Rates			
	Re	7.600%	8.485%	
	Rd		4.600%	
	Tax rate	30%		
	Target gearing	0%	30%	
	Rfree	4.0%		
	MRP	6.0%		
	Asset beta	0.60		
	beta (E)	0.60	0.75	
	beta(D)		0.10	
	wacc (vanilla)	7.60%	7.33%	
	V	$9,211	$10,111	
	wacc(classical)	7.60%	6.92%	
	V	$9,211	$10,111	Difference = pv(ITS) = $900

In this perpetuity example, we verify that the value of the ITS is just the value of the ITS capitalised in perpetuity at the cost of debt. This correct but somewhat misleading result comes about because the perpetuity assumption eliminates the risk in the ITS (either we make regular and perpetual profits with a positive ITS or we make regular and perpetual losses with a negative ITS). The logical assumption of making regular and positive profits effectively eliminates any equity risk in the ITS so it reduces to a riskless debt-derived asset valued accordingly at the cost of debt.

b. Non-perpetuity

By eliminating the obfuscating assumption of a perpetuity, we can observe the error in assuming the ITS is valued at the cost of debt.

All the base input data on costs of capital are the same as for the perpetuity case as presented above but there is a different gearing level.

DATA SERIES

	0 30/06/2004	1 30/06/2005	2 30/06/2006	… …	19 30/06/2023	20 30/06/2024
capex	($750,000)					
revenue		$275,000	$275,000	…	$275,000	$275,000
opex		($150,000)	($150,000)	…	($150,000)	($150,000)

PRIME DEPRECIATION

Book Value Open		$750,000	$712,500	…	$75,000	$37,500
Depreciation		$37,500	$37,500	…	$37,500	$37,500
Book Value Close	$750,000	$712,500	$675,000	…	$37,500	$0

RESIDUAL VALUE

Depreciation		($37,500)	($37,500)	…	($37,500)	($37,500)
Closing Book Value						$0
Sale Price						$0
Profit/Loss on sale						$0
Tax on sale						$0
NCF from sale						$0

VALUATIONS

UN-GEARED FIRM, CLASSICAL TAX						
revenue		$275,000	$275,000	…	$275,000	$275,000
opex		($150,000)	($150,000)	…	($150,000)	($150,000)
depreciation		($37,500)	($37,500)	…	($37,500)	($37,500)
residual sale						$0
taxable income		$87,500	$87,500	…	$87,500	$87,500
tax		($26,250)	($26,250)	…	($26,250)	($26,250)
Equity drawdown	($750,000)					
Cash available to Equity		$98,750	$98,750	…	$98,750	$98,750
Ungeared asset return	7.69%					
NPV (Equity)	$242,679					
Enterprise value	**$992,679**					

GEARED FIRM, CLASSICAL TAX						
Debt drawdown	($400,000)					$0
Aggregate debt	$400,000	$387,383	$374,185	…	$29,653	$0
Debt payment		($31,017)	($31,017)	…	($31,017)	($31,017)
Interest		($18,400)	($17,820)	…	($2,668)	($1,364)
Principal repayment		($12,617)	($13,198)	…	($28,349)	($29,653)
pv(Debt)	**$400,000**					
revenue		$275,000	$275,000		$275,000	$275,000
opex		($150,000)	($150,000)	…	($150,000)	($150,000)
depreciation		($37,500)	($37,500)	…	($37,500)	($37,500)
interest		($18,400)	($17,820)		($2,668)	($1,364)
taxable income		$69,100	$69,680		$84,832	$86,136
tax		($20,730)	($20,904)	…	($25,450)	($25,841)
Cash flow available to Equity		$73,253	$73,078	…	$68,533	$68,142
Equity drawdown	($350,000)					
Cash available to Equity		$73,253	$73,078	…	$68,533	$68,142
Re	8.486%					
NPV (Equity)	$329,353					
pv(Equity)	**$679,353**					
Enterprise value (V=E+D)	**$1,079,353**					
Market gearing at time=0	37.1%					

ITS		$5,520	$5,346	…	$800	$409
pv(ITS,@Rd)	**$47,624**					
ungeared TAX		($26,250)	($26,250)	…	($26,250)	($26,250)
Cost of ungeared equity	**7.69%**					
pv(ungeared TAX)	**-$263,877**					
geared TAX		($20,730)	($20,904)	…	($25,450)	($25,841)
Cost of geared equity	**8.49%**					
pv(geared TAX)	**-$210,969**					
T.D	**$120,000**					

pv(ITS) ERROR	
7.69%	Cost of ungeared equity
-$263,877	Value of tax stream @ ungeared equity
8.49%	Cost of geared equity
-$210,969	Value of tax stream @ geared equity
$52,908	Value of ITS
$47,624	pv(ITS @ Rd)
$120,000	T.D (perpetuity valuation of ITS)
$5,284	Error from wrong discount rate
$72,376	Error from perpetuity and wrong rate assumption

We observe that the standard textbook approach of valuing the ITS at the cost of debt gives an incorrect answer. If we also add in the perpetuity assumption (which also implicitly uses the cost of debt assumption), we get an even greater error.

It must be stressed that these examples use the heroic assumption of no cost of leverage. This assumption is buried in the leverage formula for the cost of capital. The standard leverage formula makes the assumption of no cost of leverage.

Assuming that there is no gain whatsoever to leverage, as in formula (4), we can approach this in two ways. The dollar value of the cost of leverage must exactly offset the apparent benefit of the ITS so we can then claim that the dollar value of the cost of leverage must be $52,908 in the above non-perpetuity example. If we wish to convert this to a cost of capital equivalent, then starting with the ungeared costs of capital, we have to *reduce* the cost of equity so that the value of the lesser geared tax payments at this reduced cost of equity exactly matches the higher ungeared tax payments at the ungeared cost of equity. If this reduction in the cost of equity at first seems, strange, recall that the benefit of the tax shield increases with the spread between the cost of debt and the cost of equity. If anything, this strange result just underscores the problems with forcing dollar effects into the discount rate. It is much better to leave them as dollar effects in the cash flow and not distort the costs of capital.

Summary

Only shareholders pay company tax. They get any benefit from the tax avoided by using debt finance. The risk in this interest tax shield is the same risk as in the equity. Profits are “shared” between share holders and the tax office (currently in the notional proportion 70:30 but after imputation credits it is effectively 81:19). Gearing effects the profits and hence the allocation to shareholders and the tax office in very similar ways. Because the risk is similar between equity and tax payments, the risk in the tax payments is similar to the risk on the equity. The ITS should be valued using a cost of equity. The value of the ITS is the difference between the tax payments valued at the cost of equity appropriate for each level of gearing.

The usual perpetuity assumption of textbooks disguises this issue by implicitly eliminating the risk in the ITS and in that case the ITS value is derived from a cost of debt. That is not a generally applicable result and so its application leads to a valuation error.

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