Australian Market Risk Premium

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Executive Summary

- We estimate the current Australian market risk premium is in the range 6.2% to 8.8% pa (mid point of 7.5% pa)

- The analysis is based on the current state of the Australian market after reviewing market data from 1875-2013.

- We analyse dividend yields and capital gains from 1882 to 2013, bond yields and inflation from 1875 to 2013. We concentrate our estimates on data from 1929 onwards.

- We include franking credits that have not been measured in total returns since 1987. The effect is an average 54 bp per annum of missing risk premium.

- We first model the market risk premium as the difference between historical market returns and the risk free rate. We do this with a variety of design parameters for the estimation process.

- We next model the implied risk premium from the excess of earnings yield over the risk free rate. We implement this model as a dividend yield model which allows the inclusion of franking credits.

- We do this for both trailing actual dividends and for forward estimates of the dividends. These estimates are provided as a national aggregate of analysts’ forecasts for earnings and dividends.

- We find acceptable reasons to believe that the MRP has changed over time.
  - The total risk in the market declined up to the GFC:
    - the volatility of the market declined off a peak in 1980 which coincided with the Resources boom from 1970 -1990,
    - the volatility of GDP (as a proxy for earnings) is declining.

  - Risk has been above average during and after the event of the GFC.
    - This is clearly evident in the implied MRP estimates.
    - The historical MRP estimates have an inverse property that shows perverse behaviour during the GFC event.
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1. Introduction

The Market Risk Premium (MRP) is the extra return investors require for holding risky assets. It plays a central role in valuation as it is a major input into the cost of capital. The required return by investors and the cost of capital are two sides of the same coin – they are the same concept viewed from the perspectives of the suppliers of capital (investors) and the users of capital (typically companies).

In order to estimate the required risk premium, we could just interrogate investors – but who do we ask - Chief Investment Officers in large pension funds, current and past individual investors, Corporate Finance Officers, future prospective investors? Even if we could identify the appropriate cohort, how do we know they are giving us reliable insight into their expectations? Surveys are conducted on these issues but they pose as many questions as they answer. For instance, does the age and experience of the respondent bias their expectations? If one has never experienced a decade of share prices going sideways (in nominal terms) then perhaps it is hard to allocate that prospect any subjective probability when deriving a personal MRP expectation. It could also be that many respondents are just reciting the MRP estimates they were taught in university finance faculties and business schools. Their personal estimates of the MRP could just be the historical estimates given the guise of personal expectations.

An alternative approach is to assume that investors base their expectations on that information they already have. In particular, we often assume that their past experience is a major driver of future expectations. This past experience has to be considered carefully. It cannot be the last few years of market returns. Individual years, particularly the last few years, demonstrate just how volatile can be the markets. Even decades of market returns can be unusually good or bad. *It is precisely the risk of experiencing these fluctuating returns for which one gets a risk premium.* We need many years of data to average out these risky outcomes and so distil the underlying excess of the risky asset return over the riskless asset return.

This long term averaging process itself introduces problems. The application of the risk premium is as a forward looking or ex-ante estimate but the historical averaging process is a backward looking, ex-post or realised estimate. Periods of poor equity return result in low realised MRP estimates which is counter-intuitive. Why would investors accept a lower risk premium because they have recently experienced low or even negative equity returns? One would imagine that the risk of prolonged negative returns make people risk averse to holding equities and so a higher risk premium would be needed to induce them to invest. This inverse problem plagues historical estimates of the MRP.
The risk premium is a premium over and above the risk-free rate. But what do we mean by the “risk-free rate”? There are so many rates as candidates for this measure that we have to be very careful and logical in what we consider to be a risk-free rate. The most common candidates are Government rates as these have (in most cases) no default risk. If held to maturity there will be no capital gain or loss events caused by interest rate changes and the bond holder will achieve the promised yield to maturity (YTM). However, there is a whole plethora of these rates with different maturities ranging from very short-dated paper (bills) to very long dated paper (bonds). This yield to maturity structure or term structure of rates is really not that important for the MRP estimate. There is no obvious term structure for equity returns. We can safely assume the expected mean return per period on the equity market is the same for short-term investments as for long term investments. There is so much noise or standard error in the estimate of average equity return that we could not hope to make fine distinctions of mean returns by investment horizon. Hence the MRP measured at the long end of the interest yield curve will typically be less than the MRP measured at the short end of the yield curve (as the yield curve normally slopes upwards). However, as long as one is consistent and combines the MRP measured at the long end with long dated risk free rates (and similarly the short end MRP estimate with bill rates) then there will not be a problem. We use the long end of the YTM curve whenever possible for estimating the MRP. That is, we use the YTM on ten year bond portfolios. The reason is a pragmatic one. There is a lot more volatility in the rates at the short end than there is in the YTM at the long end. Hence there is a lot more statistical error in estimating the MRP when using bill rates instead of bond yields.

A typical Australian government bond portfolio held by a major Australian investment fund has a duration of 3.6 years (call it 4 years for ease of mental arithmetic) so that if yields move up 25 basis points (bp) then the bond portfolio value moves down 1% or 100 bp. This example of bond capital gains or losses also elucidates a problem with using historical averages for estimating the MRP. A drop in bond rates from 6% pa to 5% pa will create a capital gain for the bond holder of 4%. Prior to the drop in rates bond holders were expecting to make 6% pa return on their bonds and now, looking ahead, they expect to make 5% pa return on their bonds. The increased return via the capital gain of 4% on top of the, now historical, expectation of 6% has not caused them to increase their return estimates. Rather, they have achieved the ex-post increased return of 4% as a consequence of the market reducing its ex-ante expectation down from 6% to 5%. If we apply the same logic to the equity market, then any reduction in the expected or ex-ante MRP will lead to an historical or ex-post capital gain in equity prices. But if we use the historical average of returns as our guide for future expectations then our estimate will move in the exactly wrong direction. The historical average MRP has increased whereas the forward looking or ex-ante risk premium has decreased. If we are using
historical capital gains or losses as a guide to the MRP then we are implicitly assuming that these gains and losses have not come about from shifts in the expected MRP.

Instead of estimating historical MRP, another approach is to find situations where the ex-ante MRP is priced in the market and attempt to extract out implied MRPs. One obvious case is the current price of shares where the prices are presumably set by rational investors whom have used their expected MRP in deciding the current value. If we knew the model they used for valuation, or at least had a model that was consistent with their model and had consistent input data, then we could extract the implied MRP from the market. The most common way this is done is to use a simple Price Earnings Model (PER) applied to the whole market. Alas, in practice it has problems— if we follow the estimator through time it would have given us negative implied MRPs and these cannot be sensibly used in forward looking valuations. Also, the PER model cannot accommodate franking credits which were introduced in 1987.

We need to be clear when estimating the MRP whether we are trying to estimate unconditional or conditional MRPs. The conditional MRP would be estimated conditional on the current state of our information, ie, the current state of our prospects. In contrast, the unconditional MRP is based on expected long term means. The easiest way to see the difference in these two measures is to think of the prospects for your football (or tennis, cricket or whatever) team winning the next grand final. These nearby prospects depend on the current state of your team versus the competition. Recent team form will be an important factor in determining your answer. However, if you are asked the prospects of your team winning the grand final in 40 years’ time, then this recent information is irrelevant. Most people fall back to the long term averages to answer this type of question. In the first case you are conditioning your answer on recent prospects whereas in the second case you are giving an unconditional estimate. The same phenomenon occurs in the MRP estimates. Some analysts are using conditional MRPs and some are using unconditional MRPs. Even though they parade as the one thing, the “MRP”, they are indeed quite different. This ambiguity is another source of confusion.

The MRP is not an equity-specific variable. It is meant to capture peoples’ estimate for the price of risk. We assume that people have the same attitude to the price of risk no matter what assets they are considering for investments. The main reason we use the broad equity market as our source of the MRP is that most assets are listed in some form or other on the equity markets and these markets are generally very liquid in that they are highly traded.

A broad market index such as the S&P/ASX 300 index in Australia or the S&P 500 index in the USA captures the valuation of a wide range of listed assets. These indices capture the top 300 listed stocks
by market capitalisation in Australia and the top 500 listed stocks in the USA. We can reasonably assume that these are of sufficient depth that they are a source for estimating the underlying MRP.

This consideration elucidates an obvious issue: the MRP estimated in this manner does not capture all risks. As it is based on a highly liquid set of traded securities (the market capitalisation weighted index portfolio), it does not have an appropriate allowance for the risk in illiquid assets – those that rarely trade. This became very obvious during the GFC event in which investors in illiquid assets that quickly needed to realise funds had a great deal of difficulty in doing so. They invariably had to sell at a significant discount which itself reflects the extra return premium required for illiquidity.

A couple of apparently arcane calculation issues are important to consider. These turn out to be fundamentally important issues but are often ignored. The two issues are first how returns are calculated and then averaged and secondly the order of taking averages in estimating the MRP. Do we calculate the average of equity returns minus the opening risk free rate or do we calculate the average of the differences between equity returns and bond yields?

The other issue for estimating averages is the order of calculating the average. First, we could calculate the equity return each period (month, quarter or year) and then subtract the opening period risk free rate. This would create a sequence of excess returns. This sequence would then be averaged over a window of given time horizon. The alternative approach is to estimate average equity returns over a span of given length and then subtract the appropriate risk free rate. If we think of the MRP as the expected risky asset return minus the expected (and also known) opening risk free rate, then under the assumption of unbiased expectations, the coming average market return is an estimator for the expected risky asset return. In this case, the coming average market return minus the opening risk free rate is an unbiased estimator of the expected MRP.

Personal taxes are totally irrelevant to the issue of the MRP. Nearly all securities trade in the band of after-company tax but before personal tax. The introduction of imputation tax has not changed this position. No more company tax is due on equities that pay franked dividends, only personal tax on the grossed up dividend so they are still traded in the same tax band of after-company tax but before personal tax. The fact that various investors in the same equity have different personal tax rates is irrelevant. If some of us (say you, the reader, a retiree and myself) buy a National Australia Bank (NAB) share at the same time for the same price, then we each get exactly the same future NAB dividends, future NAB credits and make exactly the same capital gains or losses. We face different personal tax rates so both the dividend income and capital gain/loss after-personal tax are different for each of us. But as we each paid exactly the same price for the share, the only way this can be consistent is that our different after personal tax cash flows are implicitly discounted at different after-
personal tax required returns so that we all get the same present value for the share. To introduce personal taxes when estimating required returns and performing valuation exercises, we have to consistently make the *same* personal tax adjustments to the cash flow as we do to the discount rate. In which case, nothing at all is achieved in this exercise other than to introduce unnecessary complexity. We could and do dispense with all considerations of personal tax and perform our estimates using data from the after-company tax and before personal tax band. This does not mean personal taxes are not important to each of us, it just means they are irrelevant to the MRP estimates.

The above issues and more are all investigated below. However, we first take a close look at the historical data used in this analysis. The data has been collected from many sources over the quite a few years and, unfortunately, the names of many very helpful people in the various Stock Exchanges around Australia have been long lost. I can only hope that the analysis does some justice to their generous assistance.
2. Data

In this section we examine the input data for estimating the Australian MRP. We look at the attributes of the data and consider how these attributes may impinge on the estimates of the MRP. The data we examine are stock indices, both price and accumulation, from 1875 to 2013, dividend yields from 1882 to 2013, market PER from 1961-2013, bond yields to maturity and CPI from 1875-2013 and GDP in constant prices and per capita from 1959-2012.

2.1 Price and accumulation series 1875 to 2013

Up until 1980, the Australian stock market was a set of state-based companies each running its own exchange business. Gradually, these businesses coalesced into one joint business, now called the Australian Stock Exchange (ASX). This business history is reflected in the history of the market indices. Various indices and sub-sectors were introduced over the years and in some cases back-dated calculations were created. These various indices can be located under different names – Australian Associated Stock Exchanges (AASE) indices, Commercial & Industrial index, Melbourne index and Sydney index, Lambert index etc. They are collectively plotted in Figure 1.

Figure 1: Australian equity price series 1875-2013

Apart from a couple of suspicious data points, the Lambert, AASE, Commercial & Industrial and Sydney indices all coincide for the period 1875-1936. This indicates they are all essentially the same series. Mr D. Lamberton was engaged by the Sydney Exchange for the post World War II index
revision and for the back-calculation of the Commercial & Industrial Index. So we have an historical index of industrials only. It excludes mining and finance for the period pre-1936. It is a price series only.

Accumulation series took much longer to appear than did price series. A back-dated accumulation series to 1929 is available but it is an annual series only and not shown here. This Statex Actuaries accumulation index is not plotted because there were problems with its early calculations\(^1\). Further changes have occurred recently with the index service passing to Standard & Poors, the handover taking place in April 2000. S&P have back-calculated their All Ordinaries index of 500 stocks to June 1992 and it is not the same as the ASX All Ordinaries index after 1992 up to its demise in 2000. The ASX index showed somewhat higher returns than the S&P index. We use the back-dated S&P indices, price and accumulation, from 1992.

Another way of viewing the return data is as a histogram of annual returns which we plot in Figure 2.

\[\text{Figure 2: Histogram of annual equity returns: 1929-2013}\]

\(^{1}\) Dividends were nominally accumulated during a year then added back to the index every 12 month. In a generally rising market this would result on average in understating the accumulated return. A corrected version was published by Ball and Bowers, *Australian Journal of Management* June 1987 vol. 12 no. 1 pages 1-8. The Statex index is used herein to extract dividend yields. We assume that the Statex Index has correct dividend yield data and the problem is only with the assumed dividend re-investment process.
This plot indicates the form of the distribution of returns which is approximately normally distributed but with some positive skewness; skew = 0.83. The annual average return is 11.68% pa for the entire period. Note however, that this does not include any franking credits available after 1 July 1987. We address this issue in the next section. From this distribution we see also that the chance of a negative return for any year June to June is 28% so years of positive versus negative returns occur approximately in the ratio 3:1. The annual volatility in equity returns from 1929 to 2013 is 18.8%.

2.2 Missing Franking Credits

The return on the stock market is an after company tax but before personal tax return (as are indeed nearly all returns we observe on capital markets). The MRP is usually added to a market risk-free rate and that combination is also an after company tax but before personal tax rate. So the MRP has to be constructed in a consistent manner. The ASX and now S&P add back cash dividends but not franking credits to the accumulation index. This means that the accumulation index no longer describes a simple before personal tax return on investment as taxpayers have to pay personal tax on the credits as well as the cash and capital gains already included in the indices. Franking credits are valuable which is seen when stocks go ex-dividend. On average, they fall further if franked than if unfranked. This means that investors are, on average, giving up more capital per dividend on the ex-dividend date than they did before franking credits were introduced.

We can see this in the ratio of the price index to the accumulation index. After the introduction of the imputation system the price index consistently fell further than pre-imputation so that when the cash dividend was added back to calculate the accumulation index, it was added to a relatively lower capital base because of this extra drop due to the value of the franking credits: see Figure 3. Note that the change is highly statistically different, with t-statistics of 200+ and 550+ for the ratios pre and post the introduction of imputation. The t-statistic of the difference in slopes is similarly very highly significant.
From June 1973 to June 2013, the average dividend yield on the All Ordinaries index portfolio has been 4.37% (4.97% pre-1987 and 3.98% post-1989; source IRESS). To understand the results in Figure 3, we need to carefully look at how the ASX and now S&P calculate their indices we all use for estimating MRPs.

The AOI as a price index is a linked set of movements in market capitalisation caused solely by price changes. Most indices also include dilution factors for scaling down the market capitalisation of stocks in companies for which some capital is tightly held and not deemed liquid. But this is not the case for the All Ordinaries indices; they are purely weighted by market capitalisation. The index providers attempt to adjust for any changes in corporate actions, including payments of dividends, as follows:

$$\text{AOI}_{t+2} = \text{AOI}_t \times \left( \frac{\text{MktCap}_{t+2}^{\text{close}}}{\text{MktCap}_{t+2}^{\text{open}}} \right) \left( \frac{\text{MktCap}_{t+1}^{\text{close}}}{\text{MktCap}_{t+1}^{\text{open}}} \right)$$

The market capitalisation at the open on day T+2 need not be the market capitalisation at the close of day T+1 if there has been any corporate event such as a dividend payment. Stocks go ex-dividend over night when the market is closed so the stock closes with a cum-dividend price and opens the next day with an ex-dividend price. The following sketch, Figure 4, depicts this scenario.
The stylised index calculation for one stock (with no changes in numbers of shares on issue) is

\[ AOI_{t+2} = AOI_t \times \left( \left( \frac{P_{cum}}{P_{open}} \right) \frac{P_{close}}{P_{ex}} \right) = AOI_t \times (1+R_1)(1+R_2) \quad \cdots \ (4) \]

which is precisely the sequence of returns solely due to price changes. The accumulation index is very similar except that dividends are assumed to be invested immediately the stock goes ex-dividend (but excluding the franking credits) so then that sequence becomes

\[ AOI\text{AI}_{t+2} = AOI\text{AI}_t \times \left( (1+R_2)(1+R_1) + \left( \frac{\text{Div}}{P_{ex}} \right)(1+R_2) \right) = AOI\text{AI}_t \times (1+R_1+\text{DivYld})(1+R_2) \quad \cdots \ (5) \]

If \( R_{pri} \) is the price index return and \( R_{ai} \) is the accumulation index return, then if \( R_{rel} \) designates the relative return of these two indices, across the ex-dividend event for an individual stock we have

\[ 1 + R_{rel} = \frac{1 + R_{ai}}{1 + R_{pri}} = \frac{(1+R_1+\text{DivYld})(1+R_2)}{(1+R_1)(1+R_2)} = 1 + \frac{\text{DivYld}}{(1+R_1)} \quad \cdots \ (6) \]

where the dividend yield is calculated on the ex-dividend share price. If we extend this analysis to two dividends then we get the following result:

\[ 1 + R_{rel} = 1 + \frac{\text{DivYld}_1}{(1+R_1)} + \frac{\text{DivYld}_2}{(1+R_1)(1+R_2)}. \]

Hence the relative performance after a point in time is not just a simple sum of dividend yields after that time because each dividend yield is over an invariably different capital base. Rather, relative performance after a point in time is akin to the sum of discounted dividend yields where each yield is discounted for capital gains back to the opening point in time.
The essential difference between the yields pre and post the introduction of the imputation system is that the ex-dividend price is based on the market’s capital value for future cash only for the pre-imputation period and the market’s capital value for future cash and credits in the imputation period (call it $P_{c+c}$). Then the dividend yield in the imputation period version of equation (6) can be written as

$$\text{DivYld} = \frac{\Delta \text{Div}}{P_{ex}} = \frac{\Delta \text{Div}}{P_{cum}} \times \frac{P_{cum}}{P_{c+c}}.$$  \hspace{1cm} \ldots \hspace{0.5cm} (7)

In simple terms, the relative performance across the ex-dividend date is the product of the dividend yield (based on cash dividends only, which is what the ASX and other data providers typically report) and the relative price drop-off due to the dividend and franking credit dropping out of the stock value. As credits have some capital value, relative to the cum-price, the ex-price falls further than the case of a cash only dividend so under imputation the drop-off factor is on average larger than the pre-imputation case per dollar of cash dividend. This acts to increase dividend yields based on ex dividend prices. However, this is offset against the first factor of dividend yields based on cum prices. These are lower under imputation because the share price now includes a value for all future expected franking credits. This causes a decrease in the dividend yield which is what we have observed post the introduction of imputation. This factor dominates as we see in Figure 3. This is what we expect because whereas the second factor reflects the market value of one dividend and one credit relative to the cum share price, the first factor includes the market value of all future dividends and credits. We cannot examine this issue as the case of a single drop-off factor: it is a multi-factor issue of the share price capitalising future credits, the corresponding cash dividend yield and the relative drop-off.

The current company tax rate is 30% and the current annual cash dividend yield is about 4.4% (the average from June 2012 to June 2013). This is a capitalisation weighted average yield and it implies that the fully grossed-up annual dividend yield is 6.3% which means that if all dividends were fully franked then credits would add another 1.9% annually to investors pre-personal tax income. However not all dividends are 100% franked (the capitalisation weighted average for 1988-2013 is approximately 67% franked\(^2\)) and not all credits are fully valued (we value franking credits at 43% of their theoretical full value). This means that the 1.9% must be reduced first by 67% to a value of 1.26% and then reduced further by 43% so the value added by the credits is reduced to 0.54%. Hence, the ASX and now S&P indices miss out on about 54 bp of pre-personal tax value per annum. This means that the average annual market return after company tax but before personal tax and the corresponding average MRP will be under-estimated by about 54 bp. Note that the tax rate has varied a lot over the period 1988-2013 and is now at its lowest level throughout that period. This means that

\(^2\) Estimated from our database of 19,000+ dividend, franking and drop-off events 1985-2013.
the credits were materially bigger under the higher tax rate regimes (the grossed-up values were higher). However, we can find no time variation in the value of the credits but just observe changes in the quantum of credit per dividend as the company tax rate changed. The average and most prevalent tax rate over the period was 36% which combined with the historically higher dividend yields suggests our estimate of 54 bp is on the conservative side for the period 1988-2013. This is a measurement problem only, not a theoretical problem and we do not need to derive a model to incorporate it. The problem arises solely due to the manner in which the ASX and now S&P calculate their accumulation indices. All we need do to overcome this is to add back 54 bp to the estimated MRP post-1987.

2.3  Bond yields, Australian CPI

The risk free rate we use is the nominal yield to maturity on government securities. For the earlier historical data it is YTM on trades of NSW Government paper of various maturities. For the middle part of the 20th Century it is the YTM on Federal Government bonds of various maturities and in the latter part of the data it is the YTM on purely 10 year Federal Government securities. Figure 5 exhibits this data. For the early data there were a small number of periods of no trades so the last YTM data available are carried forward. The most obvious feature is the high nominal yields for the period 1975-1995 which obviously corresponds to the period of high inflation as can be seen in the inflation data in Figure 6.

Figure 5: YTM on government securities
The CPI data in Figure 6 are annual data in the period June 1929 to June 2013. Unlike the data post-1948, the early data exhibits periods of negative inflation. This was particularly prevalent in the period prior to 1935. The period 1975-1995 was one of prolonged high inflation but the highest bout of inflation occurred in the early 1950s. This corresponded to a trebling in the world wool price (the USA Army was said to be stockpiling wool for the Korean War) which, through various mechanisms, led to a severe bout of domestic inflation.

For estimating the MRP we prefer first to estimate the return on equities over a coming period and then subtract the beginning period risk free rate. Both of these have inflation in the measures so the resulting historical market risk premium estimate, MRP(H), is a real measure. Equally, we could estimate realised real returns on equity and subtract the opening real yield on bonds. Where we do not have an accumulation index for historical data, we estimate the real total return on equity by discounting CPI out of the ASX price index and compound that real return with the nominal dividend yield. It is not material whether we extract CPI out of the capital returns or the dividend yield. As long as we use a consistent compounding equation (such as the Fisher equation) we will get consistent estimates of real return on equity. When the accumulation index is available, we discount CPI out of this accumulation index in order to estimate real returns on equity.
The real bond yields and the real equity returns are plotted in Figures 7 and 8 respectively. Whereas bonds have had prolonged bouts of negative real yields, equities have nearly always delivered real returns over any decade. This is not surprising as inflation is a cost to businesses and everyone’s costs is someone else’s revenue (this concept extends to tax collections by governments). Hence we expect equities to bounce back from inflation increases.

**Figure 7: Real Bond Yields**
The long term average real return on equities since 1929 has been 7.12% pa whereas the long term average real yield on bonds has been 2.11%. This suggests an MRP(H) estimate of 5.01% pa but this would be a poor estimate as it hides much variation in the real bond yield. Inflation is much riskier for bonds than for equities per decade so subtracting these two real measures is inadvertently introducing an asymmetric risk into the MRP estimate. We prefer to use the difference of nominal measures. Nominal prices are ones investors have actually paid with whatever foresight, if any, they had about future inflation. Using the actual future real bond yields as the expected real bond yield amounts to assuming bond investors correctly anticipated inflation and there is no compelling evidence for that proposition. Notwithstanding this observation, the high inflation of 1974 that persisted for at least a decade was unequivocally bad for both equity real returns and bond real yields. Real bond yields essentially mirror the CPI metric – high inflation equates to low real bond yields which would be consistent with bond investors not anticipating future inflation with any accuracy.
Prior to 1974, the average premium of real equity returns over real yields was a strong 7.0% pa and in any 10 year window, average real returns on equities always was higher than average real risk free rates. In contrast, after 1974 the premium has been a modest 4.0% pa there have been 10 year periods when real equity returns were less than real risk free rates.

We now turn to estimating the historical risk premium, MRP(H) using historical nominal data.
3. MRP based on Historical Estimates

When calculating historical MRP estimates, MRP(H), we can approach this in a number of ways. We can estimate the return per period (a “span” of time) on the equity market and subtract from that the appropriate risk free rate. This will generate a sequence of excess returns. These excess returns are averaged over a period of a given time length (the averaging “window”). This introduces three design parameters: the length of time over which returns are calculated, the choice of the timing of the risk free rate (at the front or the end of the equity return span) and the length of the window for averaging purposes.

The choice of these parameters invariably involves trade-offs. The longer the return span and the longer the averaging window, the less volatile will be the results. However, long spans and windows could easily hide changes in the MRP over time. There is no reason at all to assume the MRP is fixed – it is after all just a price for risk and there is no reason at all to assume that the price of risk has to be constant.

3.1 Annual MRP estimates averaged over time

The first approach we take is to calculate annual equity market total returns (capital gains/losses plus dividends reinvested) and subtract from that the 10 year bond yield at the beginning of the year - we call this the “spot” estimate of the MRP. We then average these estimates over a 20 year window. We do this for annual data (June to June) for the period 1882 to 2013. The whole period average of this estimate is an MRP(H) estimate of 5.63% pa. Figure 10 describes these estimates.

The decade-long trend in the historical MRP estimates clearly shows a strong increase into the early 1970’s from about 6% pa up to approximately 8% pa and then a long decline to around 4% pa. One has to treat historical MRP estimates with care as they suffer from the inverse relationship discussed above – low or negative realised equity returns reduces the historical MRP estimate whereas most investors presumably would regard negative equity returns as a high risk outcome and want compensation for that risk by way of higher equity risk premium.

A further estimation problem is that the standard error of these MRP estimates has increased strongly to such an extent that the t-stat for the past 40 years has averaged 1.0, that is, the standard error in the estimated MRP average is as big as the average itself – see Figure 11. We could not claim with any statistical confidence that those MRP estimates were not significantly different from a zero premium. The usual approach for overcoming this problem is to increase the sample size, which we accordingly increase to 50 year averages – see Figures 12 and 13.
Figure 10: Historical MRP estimates as 20 year averages

Figure 11: Statistics for the Historical MRP estimates as 20 year averages
Figure 12: Historical MRP estimates as 50 year averages

Figure 13: Statistics for Historical MRP estimates as 50 year averages
As we would expect, the MRP estimates based on 50 year averages are smoother than those based on the 20 year averages. The same general pattern emerges of the MRP estimate increasing up to the 1970’s and then falling thereafter. However, even when using windows of 50 years of data within each historical MRP estimate, the t-statistic of the more recent estimates has fallen below our usual definition of statistical significance of \( t\text{-stat} = 2.01 \) (N=50). Once again, we cannot claim with statistical certainty that the recent 50 year MRP estimates are significantly different from a zero premium. Obviously the cause of this outcome is the general rise in the standard error and the commensurate decline in the t-statistic towards recent years.

Using the whole period data as one single overall estimate for the MRP does not have this problem as it is highly statistically significant: see Table 1.

<table>
<thead>
<tr>
<th>Table 1: MRP point estimate 1882-2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average pa</td>
</tr>
<tr>
<td>observations</td>
</tr>
<tr>
<td>Standard deviation</td>
</tr>
<tr>
<td>Standard error</td>
</tr>
<tr>
<td>t-statistic</td>
</tr>
<tr>
<td>significance</td>
</tr>
</tbody>
</table>

While this whole period estimate is statistically very significant, it hides any time variant behaviour in the MRP. That is the compromise forced upon us by using all the data to form one point estimate.

3.2 MRP estimates based on 10 year equity returns

Instead of averaging the spot MRP estimates over a window of ten years, we can calculate the coming equity return over a span of 10 years and subtract from that the 10 year bond yield at the beginning of span period. In this model, we are effectively assuming that investors are unbiased in their estimates, namely their expected returns are the same as the average return

\[ E(R_{\text{mkt}}) = \overline{R_{\text{mkt}}} \]

so that the MRP estimate becomes

\[ \text{MRP} = E(R_{\text{mkt}}) - E(R_{\text{free}}) = \overline{R_{\text{mkt}}} - R_{\text{free}}. \]

For this estimate, we first calculate 10 year equity total returns and subtract the 10 year bond yield at the beginning of the equity return time span.
An alternative assumption is to model expectations on the recent past. If we assume people expect to make returns in the next 10 years as they have experienced in the past 10 years then the expected MRP becomes the average ten year past equity return minus the current (ie end of span) bond yield. The only practical difference between these two models is the location of the bond yield – either at the front of the equity return span or at the end of that span. We always take the current 10 year bond yield as the current expected bond yield. These two forward estimates are plotted in Figure 14.

Both estimates show similar overall time patterns. The detailed differences are due to the timing differences induced by time location of the risk free rate. If the risk free rate had been constant then both 10 year forward estimates would be the same and both would be averages of the spot estimate. When bond yields were reasonably constant (between 1882 and 1962 they were about 4% pa) we observe that the two forward MRP estimates were very similar. They diverge through the period 1970-1990 when bond yields were much more variable.

As yields on 10 year government bonds have been relatively low in recent years, the 10 year forward estimate with yields at the end of the span which has the most recent bond yield estimates corresponds with the greater recovery in the recent MRP estimates. That MRP estimate has returned to over 7% pa whereas the forward estimate with bond yields at the beginning of a span is using bond yields of 2003 which were over 1.5% higher than the current estimates. Hence those front of span MRP estimates are over 1.5% less. The latest estimates are MRP(front) = 6.2%pa and MRP(end) = 7.8% pa).

Notwithstanding these detailed differences, the trend in the MRP estimates as shown in Figures 10, 12 and 14 have a somewhat similar characteristic to the Australian market volatility as seen in Figure 15. than the end span version of the MRP estimate.

The obvious feature of Figure 15 is the initial decline in volatility up to 1930 followed by a relatively small set of cycles in volatility and then the much larger cycle beginning in the mid-1960s which peaked about 1974 then declined up to the GFC event of 2007-08. Superimposed in this general picture are three sudden increases in volatility (in 1929, 1987 and 2007) which are artefacts of the estimation process and caused by each sudden big drops in market price. These three big drops in share price enter the respective five-year volatility estimation windows then suddenly drop out as the window leaves behind that single point of large change.
Figure 14: MRP Estimates: forward 10 years

Figure 15: Australian Equity market volatility
While this plot only shows volatility of prices, the volatility of the accumulation index (if we had it that far back in history) would be almost identical because dividend payments are very stable in comparison to stock prices. This should not be confused with the volatile dividend yield series but then nearly all the volatility of dividend yield data is due to the volatile price denominator and next to nothing is due to the non-volatile numerator of the dividend amount.

The peak in the underlying trend of market volatility was around 1974-75 (apart from the October 1987 spike) which obviously coincided with turbulent times in world economies whereas the MRP peaked in 1967 and declined thereafter. This is a manifestation of the problem with backward looking historical MRP estimates. The decade 1967-1977 was the worst for share prices (excluding dividends) whereby the market share price index declined by 1.7% pa on average over these ten years – see Figure 16. Hence the backward looking MRP estimate decreases through this decade.

Figure 16: Nominal returns in share prices

Obviously 10 years of on average having capital losses would normally be regarded as a high risk event in equity investing and an expected higher risk premium would be needed to entice investors to hold shares. This is once again a manifestation of the inverse problem with backward-looking historical estimates of the MRP.
We now turn to consideration of why the MRP estimates may have declined up to the GFC event. There are two basic and possibly related reasons why there would be a decline in market-wide volatility of listed companies. Either the correlated or systematic risk between companies is declining or the cash flows (ie corporate earnings) generally are becoming less volatile. These two reasons are not automatically related. Corporate earnings could retain their volatility but become less correlated, so reducing systematic risk among companies and it is this systematic or correlated risk which cannot be diversified away in large portfolios that is often assumed to give rise to the MRP.

A market index is just a very large paper portfolio and there is some evidence for this decline in correlation in USA statistics at the portfolio level of investing. This is attributed to a structural move in the USA towards service industries that possibly have sales inherently less cross-correlated than industrial company sales. (We have no such evidence for declining correlation among Australian corporate earnings.)

The second reason for lower systematic risk is that total volatility in earnings is declining. There is indirect evidence for this in the real GDP data for Australia. The volatility of quarter-on-quarter changes in GDP has steadily declined since the 1950 – see Figure 17. This is not just a statistical artefact (which might happen if we were just getting better at measuring GDP) because exactly the same phenomenon is seen is USA GDP data. If GDP growth is a proxy for or even just correlated with company earnings growth then the decline in real GDP volatility is an indicator for the decline in the volatility of company earnings. (GDP can only be a proxy for earnings instead of sales as sales includes many costs incurred by companies which are the earnings of other sectors which are also counted in GDP so they would be double counted.)

The temporarily large size of the Resources sector of the Australian economy could very well have been a factor in the large swings in GDP over the period 1970-1990. The rise of the Service sector and the incidental eclipse of the Resource sector could very well explain the drop in GDP volatility. We must use a structural argument such as the rise of the Service sector because this phenomenon of declining volatility in GDP is observed in both the USA and Australia. Possibly service industries have less inherent correlation with each other than do industrial businesses. For example, water and electricity demand (and utilities in general) will have some income effect and price effect but they will

---

3 Campbell, Lettau, Malkiel & Xu, 2001. They found for 1962-1997 that USA stock idiosyncratic risk (company specific stock risk) increased but there was no similar trend in industry or market risk. Rather, idiosyncratic risk became an increasing proportion of total stock risk and correlations between the returns of companies declined.
not have nearly the volatility in demand as experienced by general industrial, commercial and retail businesses. Hence we expect them to have lower volatility of earnings.

**Figure 17: Volatility of Australian GDP**

In summary we can devise rational arguments that the MRP could indeed change over time. However, we can never be sure that is just ex-post rationalisation of movements we have seen in the MRP(H) estimates and these might be random deviations from an underlying true long term mean. The price of risk has to be an outcome of both the supply and demand for risk and we have no clear evidence at all that investors’ appetite for risk is rising so that they are prepared to pay more for it – ie get a lower return per unit of risk. The GFC event appeared to be a case of the reverse phenomenon. Investors had little appetite for risk so risk premia rose through that event and remain elevated.
4. Implied MRP estimates, MRP(I)

4.1 Models

A very simple constant growth dividend discount model is all we need to establish the implied MRP, which we denote by MRP(I). For such a model applied to an individual stock with a risk premium of RP we have

\[ \text{Value}_t = \frac{\text{Div}_{t+1}}{R_{\text{free}} + \text{RP} - g}. \]

In this model, \( g \) is the expected growth (assumed constant) in the expected dividend per share. It is not the natural growth in the business per se. That would be the growth in the total earnings of the business, maybe driven by population growth and productivity gains and so correlated with GDP per capita. If a company paid out all of its earnings then there would be no growth in dividends per share. Instead, there would be growth in shares on issue as the company issued more equity in order to acquire capital. This is exactly what would happen if the company offered a dividend reinvestment plan (DRP) and all shareholders elected to accept all dividends via such DRPs. But as we have seen above, the index providers cater for these changes in the capital base with their daily index calculations so we need not concern ourselves with that issue for past data.

Moreover, assuming any future distributions are made at fair value, we do not have to model future capital issues as the present value of that stream of future cash and capital distributions should be the same as the present value of the dividend-only stream. If this was not the case then we could hardly claim to have a model in perpetual equilibrium.

The following sketch, Figure 18, is the logical cash flow for the dividend valuation model. Note that the value at time T+1 is just \((1+g)\) times the value at time T because the future dividends at time T+1 are just \((1+g)\) times the future dividends at time T.

*Figure 18: Schematic of valuation cash flows*
To calculate the implied risk premium (RP) arising from the current market prices, we make the model value meet the market price, i.e. we set \( \text{Value}_t = P_t \).

The fair value at time \( T+1 \) compared with the fair value at time \( T \) is just the present value of the perpetual stream of dividends in which all have grown by the factor \( 1+g \). Hence the fair value at \( T=1 \) is just \( (1+g) \) times the fair value at time \( T \).

The valuation formula used here is one for the ex-dividend cash flow – that is, the value in the above formula is the value of all future dividends, not including the current dividend. This means that the expected return on equity for a period \( T \) to \( T+1 \) is then given by the growth in price, \( g \), plus the dividend yield,

\[
\text{Return}_T = \frac{P_{T+1} + \text{Div}_{T+1} - P_T}{P_T} = \frac{(1+g)P_T + \text{Div}_{T+1} - P_T}{P_T} = g + \text{DivYld}_T. \tag{8}
\]

In other words, the expected return on equity is the sum of the expected dividend yield and expected capital growth which in a constant dividend discount model is the same as expected dividend growth. Whilst we often refer to “growth” in the Dividend Discount Model (DDM) as the growth in the dividend, for a constant dividend growth model it is also the growth in the share price. This is an important point for calibrating the model to market data.

Upon re-arranging the formula above, we get

\[
P_T = \frac{\text{Div}_{T+1} \times R_{\text{free}} + RP - g}{\text{Div}_{T+1} \times R_{\text{free}} + RP - g}
\]

\[
1 = \frac{\text{DivYld}_{T+1}}{R_{\text{free}} + RP - g}
\]

Solving for the implied risk premium we get

\[
RP(I) = \text{DivYld}_{T+1} + g - R_{\text{free}}
\]

which is just a re-arrangement of the statement that:
Return on Equity = \(\text{DivYield}_{t+1} + g = R_P(I) + R_{\text{free}}\).

In equilibrium we expect growth in earnings per share and growth in dividends per share to match the return on equity from the retained earnings. Shareholders ought to be indifferent (from a valuation perspective) between receiving dividends now and leaving the capital in the firm as retained earnings where it then earns its cost of capital. In this manner the present value of a future and larger dividend matches the current value of an immediate dividend. This is a quite different issue to some investors, particularly retirees, wanting dividends as a source of income. The choice between dividends and capital gains is an issue of a clientele effect for shares, not a valuation issue. Franking credits need not be material to this indifference argument as a firm can operate a dividend reinvestment plan (DRP) which has the effect of distributing franking credits to shareholders and retaining the earnings in the firm.

In conclusion, we must have growth matching the return on equity for the retained earnings,

\[ g = \text{Re} \text{(retained\%)} = (R_{\text{free}} + R_P)\text{(retained\%)} \]

where Re is the return on equity (which, in turn, comprises the sum of the risk free rate and the risk premium) and retained\% is the proportion of earnings not paid out to shareholders as dividends. Any deviation from this in equilibrium (ie in perpetuity) means that eventually the company will destroy all value (if growth is less than required) or will create infinite wealth (if growth is higher than required). These are the artefacts of the assumed constants in this dividend growth model.

When applied to the whole market, the risk premium, RP, becomes the market risk premium (MRP) and g becomes the capital growth of the market.

Combining the equilibrium growth formula with the RP formula and applying it to the whole market, we have

\[ MRP = \text{DivYield}_{t+1} + (R_{\text{free}} + MRP)\text{(retained\%)} - R_{\text{free}} \]

\[ MRP(1 - \text{retained\%}) = \text{DivYield}_{t+1} + R_{\text{free}}\text{(retained\%)} - 1 \]

\[ MRP = \frac{\text{DivYield}_{t+1}}{(1 - \text{retained\%})} - R_{\text{free}} \]

\[ MRP = \text{EarnYield}_{t+1} - R_{\text{free}} \]
In summary, the implied market risk premium is the whole of market ex-ante earnings yield minus the prevailing market risk free rate.

So there are two models that appear equivalent:

1. The dividend yield model:
   \[ MRP(I) = DivYield_{t+1} + g - R_{free} \]  
   \( \ldots \) (9)

2. The expected earnings yield model
   \[ MRP(I) = EarnYield_{t+1} - R_{free} \]  
   \( \ldots \) (10)

In both cases, all data are applied to the whole stock market as a single equity portfolio. We can use either historical data or forecasts for dividends and earnings.

### 4.2 Including Franking Credits

Franking credits are attached to dividends, not earnings. Presumably when dividend imputation was introduced into Australia it caused a rise in the value of stocks without any apparent increase in after company tax earnings. This would happen because shareholders valued the new franking credit stream as an additional benefit of owning stocks and it must have resulted in an increase (possibly a one-off) in the Price-Earnings ratio (PER) for Australian stocks. The introduction of imputation by itself should not have caused any change in corporate earnings. Imputation is a reduction in the total tax paid by the ultimate owner of the business, the shareholders. Its introduction reduced the imposition of double taxation on company profits distributed as dividends i.e. first company tax and then personal tax. Shareholders’ total tax on corporate income was reduced as they received a credit for tax already paid by the company. Hence the only way to see this effect in the earnings version of the model is (was) by an increase in the PER.

An increase in the price for no apparent increase in the after-tax earnings implies a decrease in the earnings yield. Hence the earnings yield model would erroneously conclude that the introduction of imputation reduced the MRP as per equation (10) when such a conclusion is irrational. Imputation has no effect on risk – it is a cash flow effect only. The dividend yield model on the other hand has an automatic correction process built in. If dividend income rose by the addition of franking credits and shareholders valued this additional stream then share price would have risen. But then the increase in dividend income is offset by the increase in share price. If we only include the market value of these credits (the extra value reflected in the price increase, not the full face value of a credit) then we will
fully offset the increase in dividends with the increase in share price and the dividend yield with imputation will not have changed and the MRP estimate from equation (9) will not be changed by the introduction of imputation. That is precisely how it should have been as imputation should not have changed the risk characteristic of corporate after-tax earnings.

In summary, the DDM approach allows all of the changes in the imputation system to be explicitly included in the franking credits paid along with the cash dividends. In contrast, the earnings yield model (or equivalently, the PE model) can only implicitly recognise credits via changes in the PE ratios which are very difficult to detect. In a forward-looking PE model, we are left trying to detect changes in the future PE ratios of analysts that arise from their franking credit assumptions and this task may be all but impossible.

The question of how to include franking credits becomes a question of consistency. The “price” data being used in the analysis is taken from market prices. The design concept of the analysis is to extract out the implied market risk premium from the market data. If we denote the market value of a credit by theta ($\theta$) then the appropriate theta to use in the analysis is that value which market participants collectively use in determining their price level for all equities. The relationship can be expressed as the following in which theta has a value $0 \leq \theta \leq 1$.

$$P_t(\theta) = \frac{Div_{int}(\theta)}{R_{free} + MRP - g}$$

Theta is a concept developed for a franked dividend of a single share. Any dividend of an individual share can only be fully franked or unfranked. There is no legal concept of a part-franked dividend. People usually employ the words “part-franked dividend” to describe the net effect of a bundle of some fully franked and some unfranked dividends. Theta is the measure of the market value of the franking credit of a fully franked dividend. However, when considering the whole market as one big share, the whole-of-market franking credit will be a partly franked dividend, made up of the blend of fully and unfranked dividends. The theta value then must be a multiplication of the level of the franking of the aggregate dividend and the market value of a fully franked dividend. This is the net theta, which then allows for the extent of franking credits to the generic market dividend.

From our database of 19,000 dividend events from 1985 to 2013, we estimate the market capitalisation weighted average franking level of approximately 67%. Our estimate for the average value of a generic (ie market-wide) franking credit is 43% of the face value of a credit.
For a typical dividend cash yield of 4% that is 67% franked with those franking credits valued at 43 cents in the dollar, the value of the 4% yield as a grossed up yield is 4.49% cash plus credit (under a 30% company tax rate.) The logic is as follows:

\[
\begin{align*}
4\% \text{ cash fully franked} & \quad = \quad 4\%/(0.70) \quad = \quad 5.71\% \text{ grossed up dividend} \\
\text{fully franked credit} & \quad = \quad 5.71\% - 4\% \quad = \quad 1.71\% \\
67\% \text{ franked credit} & \quad = \quad (0.67) \times 1.71\% \quad = \quad 1.15\% \\
\text{market value of partial credit} & \quad = \quad (0.43) \times 1.15\% \quad = \quad 0.49\% \\
& \quad = \quad (0.43) \times (0.67) \times 1.71\% \quad = \quad 0.49\% \\
& \quad = \quad (0.29) \times 1.71\% \quad = \quad 0.49\% \\
\text{value of the cash plus credit} & \quad = \quad 4\% + 0.49\% \quad = \quad 4.49\%
\end{align*}
\]

The net theta (0.29) is then the product of the franking proportion (0.67) and the theta per credit of a fully franked dividend (0.43). The gross-up rate from a cash dividend to a franked dividend is then calculated, under a 30% company tax rate, as $1.1235 = 1 + 0.29 \times (0.30/0.70)$. This estimate for stock exchange traded events is substantially below the estimate from taxation statistics in which the average franking level of all reported dividends is 90% (calculated from the latest reported ATO data for 2011). This is not surprising as the ATO data include many private companies which pay a higher proportion of fully value franked dividends. For the purposes at hand, the estimate from listed equities is more appropriate so we consider an estimate for franking of 68% to be the preferable one.

### 4.3 Historical Dividends and Earnings yields

Dividend yields have varied a lot over the years though this variation essentially mirrors price volatility – dividend yields generally move inverted to price. High price corresponds typically to low yield and low price typically corresponds to high yield. Figure 19 plots this data from 1882. This series is a composite of three series. The early data is due to Lamberton ("Some Statistics Of Security Prices In The Sydney Market (Annual Averages) 1875-1955"), the mid series is the Statex dividend yields and the latter part is the ASX All Ordinaries Dividend Yield series. The Lamberton and Statex data overlap for the period 1929 – 1955 and for that overlap period the Statex dividend yields average just 75% of the Lamberton yields. This discrepancy is probably mostly explained by Lamberton calculating simple arithmetic averages and not weighted averages. This would exaggerate the impact of high yielding but small capitalisation stocks. In addition, stocks that did not pay a dividend would not be in Lamberton’s average so this would also bias the average upwards. We have applied a 25% discount factor to the Lamberton dividend yields for the period 1882-1928 before combining the data into a single series from 1882 to 2013. So as not to derive estimates based on uncertain historical dividend yields, we only estimate the implied MRP model for the period 1929-2013.
The whole period average of these dividend yields 4.74% pa and post 1928 it is 4.37% pa. It has been 4.26% pa over the most recent decade. Unfortunately we do not have the equivalent historical earnings data as far back as the dividend data. As per Figure 20, we have it back to only 1974.

**Figure 20: Earnings and Dividend yields**
We do not implement here the earnings yield version of the MRP because of the two issues of lack of historical earnings data and a lack of being able to include franking credits after 1 July 1987. Rather, we implement the dividend yield version of the model.

4.4 Appropriate Growth Rate

There are three inputs into the model: expected dividend yields (including franking credits where relevant), expected capital growth and expected risk free rates. Dividend yields are always real quantities so we need growth and risk free rates also in consistent metrics: either both nominal measures (in which case expected inflation is in both but cancels upon subtracting the difference) or both real. From Figure 16 above we see that long term nominal growth in the share price only (excluding reinvested dividends) was 7.04% pa. The inputs to the model require the expected capital growth and as usual we take the average to be the expected estimate. This is not the future CAGR but the future arithmetic average. We examine this detail in the Appendix. Note that we do not have to worry about capital issues diluting these past data as the index providers have carefully allowed for those corporate events. Recall from equation (3) above in Section 2.2 that the index calculation is a price only index as its calculation adjusts for any capital changes. Hence the growth in the share price index is indeed the growth in capital alone, not modified by any growth in capital subscribed to or paid out of the market.

A further requirement of the growth factor is seen in equation (8) in that the expected return on the market must equal the sum of the expected growth rate and the expected dividend yield. For the entire period 1929-2013 the average annual return (excluding franking credits) on the market has been 11.68% pa. Also over this period we have seen that the cash only dividend yield has averaged 4.37 pa.

The appropriate growth, g, rate must then satisfy the requirement, as given in equation (8), of

\[11.68\% = 4.37\% + g\]

which solves for \(g = 7.31\%\) pa. However, as the 11.68% average return results from an accumulation index calculation, it includes a compounding effect of capital growth on the dividend yields. Allowing for this compounding effect, the solution for the growth parameter is 7.00% pa. This is very close to the actual average price appreciation which we have seen averaged 7.04% pa from 1929 to 2013.

The naïve MRP(I) estimates adds the average compound average growth rate of 7.04% to the dividend yield and subtracts the risk free rate. This produces the following MRP estimates – see Figure 21.
This model has the unfortunate property of producing a long period of negative risk premiums. Plotting the three components of the model makes it very obvious why this occurs – see Figure 22.
The high risk free rate causes the negative MRP estimates. Whereas dividend yields hardly change at all with inflation, the high nominal risk free rates in the late 1970s and early 1980s which coincide with high inflation is the cause of these associated MRP(I) estimates becoming negative. This is the result of an inappropriate mix of parameters. The growth parameter in model (9) is a long term estimate based on 84 years of capital growth – the growth parameter in the dividend yield model is assumed constant.\footnote{A more elaborate model could use variable estimates over an initial period and then a perpetual model for the tail. This same problem would emerge in the tail perpetuity so using such a more elaborate model does not resolve or eliminate the issue.} This growth in dividends must be the same as the long term growth in the share price for this model. However, the risk free rate is a spot rate which responds to current circumstances, including expected inflation. The problem then is the mix of a constant growth rate with a variable risk free rate.

Two approaches can be used to address this mismatch. We can either make the growth rate a variable one along the lines of the variable spot risk free rate or we can make both the growth rate and risk free rates long term expected averages. Because the model only requires the difference between the expected growth rate and the expected risk free rate, we can calculate this difference based on real rates and eliminate any measure of expected inflation. Dividend yields are always real as they are calculated as nominal dividends divided by nominal share price which always results in a real metric.

The long term average dividend yield is 4.37% pa (see Figure 16). The long term real return on equities, including dividend yield, is 7.12% pa (see Figure 8) so the implied long term real growth in capital is 2.75% pa. The long term average real bond yield is 2.11% pa (see Figure 7). Hence the long term MRP(I) estimate based on real parameters is 5.01%. This is just a bit lower than the average of the nominal estimates which is 5.09% pa.

As per the alternative of making the expected growth rate a variable, this would require matching the growth rate to that observed in the equity price series. This is required in order to meet the restriction of the model that expected growth in dividends equals expected growth in share price. Figure 16 demonstrates the average growth in the share price over spans of ten years. If we take the actual averages for each ten year period as a measure of expectations (i.e. investors are unbiased in their expectations over ten year periods) then the resulting MRP(I) estimates are as follows: see Figure 23. These data are plotted at the end of each 10 year window.
The long term average MRP(I) based on this analysis 7.19% pa. After the period of high inflation which was bad for share prices in the medium term, the MRP(I) estimate recovered to move approximately within the band 5%-10% pa for the most recent 20 years. The average MRP for these past 20 years has been 8.81% pa.

4.5 Implied MRP estimates from forecast data

We now use analysts’ forecast data in place of historical data. We observe that they demonstrate the same pattern in their forecasts of earnings being highly correlated with their estimates of cash flow or dividends: see Figure 24. This is important as they usually only report estimates for long term growth in earnings but not growth in dividends. It appears reasonable then to accept their estimates of growth in earnings as a good proxy for growth in cash flow and growth in dividends.
In the above plot, “CPS” means cash flow per share, “EPS” means earnings per share and “DPS” means dividends per share. All forecasts for all countries are expressed in USD and aggregated to a world amount as a weighted sum, using the weights in the MSCI World Index. For example, in February 1999 the aggregate cash flow per share was US$94.22, the aggregate earnings per share were US$49.92 and the aggregate dividend per share was US$19.87.

Turning to the Australian data, the following plot (Figure 25) is the aggregate analysts’ forecasts for the next 12 months for cash flow, earnings and dividends.
Figure 25: Analysts’ forecasts for Australia for next 12 months

We consistently use end of month data: we use analysts’ forecast as they exist at the end of a month, we use market share prices at the end of a month and Commonwealth Government 10 year bond yields at the end of a month.

The tie between the two models based on dividends and earnings is the payout variable, as seen in the two connecting relationships between the dividend and the earnings models:

\[
\begin{align*}
\text{DPS} &= \text{Payout} \times \text{EPS} \\
\text{Growth} &= \text{Re} \times (1-\text{Payout})
\end{align*}
\]

These relationships were used to move from the dividend model to the earnings model\(^5\). The assumed payout ratio then becomes important in connecting the two models. The implied and actual estimates of payout ratios are as follows:

\(^5\) Note that dividing Price through the first equation converts it to a yield model as Dividend Yield = PayOut * Earnings Yield and it in this yield form that we apply the model.
Analysts appear both mildly more conservative and more stable in their estimates of payout ratios than the actual experience. On average, the earnings payout ratio of dividends was actually 61%, whereas the analysts expected an average payout ratio of 60%. Even though these averages are very close, they disguise aspects of the differences which are that analyst are below the actuals in 80% of all months. We would not expect to see such a bias built into analysts’ forecasts. The could recast “payout” away from cash dividends paid out of accounting earnings and expressing it as cash dividends paid out of cash income from operations. One has to have the cash income in order to pay it out as a cash dividend – especially in perpetuity models such as the simple ones under review. We can all think of recent examples where this simple rule was violated with the inevitable eventual demise of the offending business.

The dividend payout ratios based on Cash Flow from Operations per share (CPS) are even more aligned as we see in the following: see Figure 27.
Once again, the forecasts are very close to the actuals and analysts again tend to err on the conservative side: in 84% of months their forecast payouts are below the actuals. There is very modest apparent bias in their payout forecasts using either earnings or cash.\textsuperscript{6}

Estimates of payout ratios are not helpful in distinguishing between a cash-based model using DPS and an earnings-based model using EPS. The choice of the model comes back to the essential feature that franking credits can be incorporated easily into the dividend model but with difficulty in the earnings model.

Note that these estimates of actual payout ratios will differ somewhat than those estimated for the general Australian and World markets. This is because the estimates of actual payments are only calculated for the same portfolio of stocks for which analysts are supplying forecasts. We use the MSCI World index as the base portfolio and the MSCI Australian index as the Australian constituent of that World Index. If there are stocks in the MSCI Australian index for which analysts are not

\textsuperscript{6} In a previous version of this analysis we concluded that analysts had reasonable forecasts of cash payout ratios but biased forecasts of earnings payout ratios. We now know that the forecast data and actuals data were not collected on the same basis which threw up a spurious discrepancy in payout ratios.
supplying forecasts then the suppliers of the databases of forecasts typically re-weight the reduced portfolio by proportionally increasing the stocks with active forecast data and zero weighting those with no forecast data.

With these data, we can now form estimates of grossed-up dividend yields – see Figure 28. We gross up by applying a net theta factor = 0.29 which the product of the franking proportion estimate (0.67) and the market value per credit of a fully franked dividend (0.43). We use the prevailing tax rate each year for the gross-up factor which under a tax rate of 30% results in a gross-up factor = 1.1243 and this is calculated as 1+0.43*(0.30/0.70). We can find no time variation in the value of a fully franked credit so we use a constant theta value of 0.43 throughout. For most of the period under consideration here, the company tax rate has been 30% so the gross-up factor has been 1.124 for most of the time.

**Figure 28: Grossed-up dividend yield forecasts**

The other major input factor we need is the estimate for the growth in the dividends per share. We can get this from a variety of sources. First we can directly use the estimates that analyst provide for “Long Term Growth” in EPS. Because their forecasts of payout ratios are reasonably constant (see Figure 26) we can safely apply their forecasts of growth in EPS to growth in DPS. Secondly, we can directly estimate the growth over time in their forecasts of EPS compare this to a third measure, the
growth in the actual EPS data. And we can compare all these estimates against the actual long term average capital gain estimate for 1929-2013 which has been 7.04% pa. Recall that in the Constant Growth Model the total return on equity is the sum of dividend yield and growth and as total return is always the aggregate of dividend yield and capital gain, it must be that the growth factor is just another description of capital gains.

The three estimates based on forecast data for the period February 1999 to June 2013 are plotted in Figure 29. It is immediately apparent that the Long Term Growth forecasts of analysts are significantly biased high when compared to the growth in actual EPS. And quite paradoxically, the realised growth in their forecasts is on average very close to the actual growth in EPS. Hence, analysts as a group do not make biased forecasts on average for actual EPS but they do forecast quite optimistic growth rates. And just to underline the confidence in these data, the forecasts and the actuals have grown at an average rate for these last 13-14 years at rates reasonably close to the 1929-2013 average growth of 7.04% pa and also close to the requirement of the model that the sum of growth and dividend yield equals the expected return on the market which we have seen above results in a requirement that the expected growth rate should be 7.00% pa (where expectations of returns and yields are based on historical averages).

Figure 29: Growth rates of Forecasts and Actuals
In order to form MRP estimates based on these analysts’ data, we use the realised long term growth rate of 7.04% pa. Following equation (9), we take the analysts’ expected dividend yields, grossed up for net franking credits using a net theta value of 0.29, add to that yield the long term growth rate and subtract the risk free rate. The results are plotted in Figure 30. We note that there is some offsetting estimation happening in the analysts’ dividend yield forecast and the risk free rate. This produces a less volatile forecast for implied equity return.

![Figure 30: MRP implied by analysts' forecasts](image)

The estimates of the implied MRP are on average quite low prior to December 2001 whereby they had averaged just 3.4% pa. The Tech Bubble was particularly harsh on any metric based on dividend yields: share prices had risen up to this period but analysts had not forecast a commensurate increase in cash flow, let alone increases in dividends. From January 2002 to June 2008, the analysts’ implied MRP averaged 5.9% pa which was very close to the 130 year historical average of 5.7% pa as seen above in Table 1. During the GFC, implied yields were very high and the commensurate risk premiums were very high. After the GFC, implied risk premiums never settled back to their pre-GFC level, only temporarily dropping back to 7% pa and then began to climb again, to the extent that the latest implied risk premium is 8.8% as at June 2013. As a group, analysts are seeing the markets as significantly more risky in the future than has been normal in the recent past.
5 Comparing the Models

Superficially, the two models of estimating the MRP, one which we call MRP(H) based on estimating historical differences between market returns and risk free rates, the other which we call MRP(I) based on dividend yields plus growth minus risk free rates, appear to be quite different models. However, once we realise that the growth factor in the dividend yield model is just the same as the capital return on the equity, we see they are essentially the same model:

\[ \text{MRP(H)} = \text{Return on market} - \text{Risk free rate} \]

\[ \text{MRP(I)} = \text{Dividend Yield} + \text{Growth} - \text{Risk free rate} \]

\[ = \text{Return on Market} - \text{Risk free rate}. \]

Each version of the model has its merits. The historical version, MRP(H), gives an unequivocal view of what the MRP has been in the past. However, it has the disadvantage of having an inverse behaviour of the MRP estimate decreasing as the equity markets realise low or even negative returns. Periods of heightened risk with severe negative returns one would normally regard as periods when investors would need to be enticed with higher risk premiums, not lower risk premiums. The recent GFC has been a strong example of this type of event. In the six years from June 2007 to June 2013 the historical annual risk premium has averaged -3.83% pa because the market returns were so poor. Obviously six years is too short a period over which to form an estimate but it does highlight the problem. However, this overlaps with another problem of the historical MRP estimates: the volatility in the annual estimates is quite high so that many years of data are needed in order to create reliable estimates from averages. This long term averaging then disguises any recent movements in the MRP such as for the GFC event.

The volatility in the estimates comes from the volatility in share returns which is in the order of 19% pa and the volatility of the annual risk premiums is approximately 16% pa. But the estimates of the market risk premiums are in the order of 4%-8%. An average calculated from a sample of 50 years data has a standard error of approximately 2.3% which means the estimate of the mean annual risk premium has a disturbingly wide confidence range of ±4.6%. Even a sample of 100 years has a confidence range of ±3.2%. This problem is inherent in using a model based on stock returns which are quite volatile data.

The dividend yield version, MRP(I), of the risk premium estimates has the merits that it can be used in both an historical form using past dividend yields and past growth rates to form historical MRP
estimates as well as used in a forward-looking form whereby forecasts by analysts around the world can be distilled to obtain forecasts of the implied MRP. We can follow these forecasts through time and estimate the MRP implied from those forecasts. The MRP(I) estimate has the attribute that analysts tend to counter movements in the risk free rates with offsetting movements in their implied dividend yields. Then the volatility in their implied MRP estimates is essentially the volatility in the risk free yield that is subtracted from the implied return on equity. The volatility of the risk free rates, 3% pa, is much less than the volatility of the stock market returns, 18% pa. Hence the estimates derived from the forward looking implied model, MRP(I), are much less volatile than those derived from the direct historical model, MRP(H).

The historical version of the MRP(I) model does have a similar problem as the MRP(H) model in which high risk free rates tend to produce low or even negative MRP estimates. The only practical solution is the same in all cases: use long term averages either by averaging the volatile annual estimates or using long term averages in each of the parameters and form market risk premium estimates as the differences of these long term averages.
6 Summary

We have examined the market risk premium from a variety of approaches. The first approach is the usual one of directly estimating the MRP as the difference between historical market returns and the risk free rate. We used annual Australian stock market data from 1875 to 2013 in order to form these estimates. The quality of the data has increased significantly over the years. The early data was purely price data and then did not cover all segments of the market. This early data paraded under a few different names but when we plotted them all we observed that apart from a few isolated data points, they were all essentially the same data series. The early data prior to 1929 was essentially due to a backdating exercise conducted for the Stock Exchange by Mr D. Lambert in 1955. He also derived a dividend yield series for this period but when we compared it where it overlapped with to a later series dating from 1929 we found the Lambert series over-stated the dividend yields on stocks. Hence we only formed our historical MRP estimates on data from 1929 to 2013. The data from 1929 onwards was meant to be an accumulation series but problems with its design meant dividends were not re-invested immediately they were paid. The resulting calculated series somewhat understates the return on the market by the amount of the missed market return on the dividend capital. However that is likely to be a quite small amount in comparison to the range of uncertainty in the MRP estimates so it is not a significant problem.

Good accumulation series became available for the ASX from 1979 onwards and we make use of that data. However all such data does not include a significant source of investor return which emerged with the introduction of the imputation tax system. Cash dividends are assumed to be immediately re-invested in the market but franking credits are ignored by the index providers. Shareholders do not ignore them though and they are priced into the market value of shares. We estimate that about 54 bp of pre-personal tax return per annum is missed by ignoring franking credits. We add this back into market returns for the purposes of estimating historical MRP estimates post 1987.

We formed annual estimates of the MRP using these data and then averaged the annual MRP estimates over 20 year windows. When examining the statistics of these estimates we found many after 1970 were statistically quite insignificant. To overcome this problem, we extended the averaging window to 50 years. All of these 50 year averages produced a long term average of 6.1% pa but the latest estimates was just 4.7% pa. However, because of the extended averaging window this one data estimate encompassed the years 1963-2013 so a lot of smoothing of potentially quite different epochs of the market is likely with this analysis. Most definitely it will miss any increased risk arising from the GFC event.
Instead of averaging annual risk premiums over windows of various lengths, we turned to estimating the average equity return over a span of 10 years. The risk free rate to be subtracted off that return in order to estimate MRP can be either located at the beginning of the 10 year span or at the end. The difference is one of assuming how expectations are formed. If at the end then we are assuming that expectations of equity returns are estimated by the average of the past 10 years of market returns. As the current (ie at end of a 10 year span) risk free rate is the yield expected for the next 10 years then the difference between these two is an estimation for the MRP. If we place the risk free yield at the beginning of a 10 year equity return span, we are assuming that investors are unbiased in their estimation of future equity returns and as the current (ie at beginning of a 10 year span) risk free rate is the yield expected for the next 10 years then the difference between these two is also an estimation for the MRP. The latest 10 year MRP estimate of the first model (yield at the end) is 7.8% pa and the latest 10 year MRP estimate of the second model (yield at the beginning) is 6.2% pa.

The alternative model of implied MRP estimates was estimated next. This model is based on a constant growth rate for a dividend discount model. The growth rate is modelled as the growth in the dividend amount but it is also the growth rate in the ex-dividend share price. The difference between the two models is superficial. Both are models of market returns minus the risk free rate. Whereas the dividend yield model builds the market return from dividend yields plus capital growth, the historical model directly uses realised market returns.

The yield version of the model can be applied to either historical yields or forecast yields. This has the advantage if elucidating the view of current and future risk as against an historical and long term average risk using past market returns. It is also straightforward to include franking credits with the dividend yield model.

Applying this model with an historically constant growth rate and prevailing dividend yields minus prevailing risk free rates delivered a series of implied MRP estimates that were negative for a long period which corresponded to high risk free yields which in turn we due to high inflation. We overcome this problem by using variable growth rates. At each point in time the model is calibrated using the growth rate in capital of the past 10 years. This model produced an estimate which averaged 7.19% pa since 1929 with the latest estimate being 8.06% pa.

The advantage of the yield model becomes apparent when we can apply it to forecasts. There are extensive databases of analysts’ forecasts of earnings and dividends for a vast number of companies and these can be aggregated into portfolio forecasts, including market index portfolios. We analysed
their forecasts for Australia and found that their average growth in past forecasts very closely matched the growth in actual data. However, their explicit forecasts of growth were much more optimistic so we did not use those growth forecast. Instead, we used actual growth rates and not forecasted growth in order to implement the model.

The estimates of the implied MRP quite low prior to December 2001 but reverted to near the long term historical mean from January 2002 to June 2008. Analysts’ implied MRP averaged 5.9% pa which was very close to the 130 year historical average of 5.7% pa. During the GFC, implied yields were very high and the commensurate risk premiums were very high. After the GFC, implied risk premiums temporarily dropped back to 7% pa and then began to climb again. The latest implied risk premium is 8.8% as at June 2013.
7. References and readings


Officer, R.R 1989, *Rates of Return to Shares, Bond Yields and Inflation Rates: an Historical Perspective*, in R. Ball et al. (eds) Share Markets and portfolio Theory, 2ed, QUP.


Appendix

Expectations should be based on expected arithmetic averages

The correct calculation method for the expected MRP is the arithmetic average, not the CAGR nor the continuous average return. The reason is quite simple. When examining historical returns of bonds and equities we effectively are calculating a performance analysis of the indices (and also of fund managers following an index replication strategy). The conclusion is that the past performance of the index or the fund ought to be reported as CAGR (or continuous returns, denoted by $R^C$) and not as historical arithmetic averages, (denoted by $R^D$). However, for estimating the MRP and generally calculating proxies for expected metrics, we wish to take a forward view of returns, not an historical view. The past performance of the market represents one sequence of outcomes which could have happened from a whole plethora of possible outcomes. The following diagram is a stylised sketch of a set of future market possibilities. Each period return is independent of the previous and the market may rise by 20% or fall by 10% with equal probabilities. The expected return at each period is thus 5% calculated as the probability weighted sum of $0.5 \times 20\% + 0.5 \times (-10\%) = 5\%$. At each point in the array, the realised average returns are calculated and, combined with the probabilities, the expected average returns across the distribution of outcomes are then calculated.

Figure 31: Why we use the Arithmetic Average returns

<table>
<thead>
<tr>
<th>Return discrete</th>
<th>up 20%</th>
<th>-10%</th>
<th>down 10%</th>
<th>0.50 20.00% 18.23%</th>
<th>-10.00%</th>
<th>-10.00%</th>
<th>-10.00%</th>
<th>-10.00%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return continuous</td>
<td>up 18.2%</td>
<td>-10.54%</td>
<td>down 50%</td>
<td>0.50 20.00% 3.85%</td>
<td>-10.00%</td>
<td>-10.00%</td>
<td>-10.00%</td>
<td>-10.00%</td>
</tr>
<tr>
<td>Probability</td>
<td>50%</td>
<td>50%</td>
<td>-10.00%</td>
<td>0.25 18.23%</td>
<td>3.85%</td>
<td>3.85%</td>
<td>3.85%</td>
<td>3.85%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Value from compounding the average</th>
<th>Value</th>
<th>Expected Arithmetic average</th>
<th>5.00%</th>
<th>5.00%</th>
<th>5.00%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compounded Arithmetic average</td>
<td>$105.00</td>
<td>5.00%</td>
<td>109.12</td>
<td>5.00%</td>
<td>113.40</td>
</tr>
<tr>
<td>Compounded CAGR</td>
<td>$105.00</td>
<td>4.46%</td>
<td>109.12</td>
<td>4.46%</td>
<td>113.40</td>
</tr>
<tr>
<td>Compounded continuous</td>
<td>$103.92</td>
<td>3.85%</td>
<td>108.00</td>
<td>3.85%</td>
<td>112.24</td>
</tr>
</tbody>
</table>
The expected value at time T=3 is the probability weighted sum of the possible outcomes

\[ \text{Exp Value} = \sum \text{prob}_i \times \text{Outcome}_i \]

\[ = (0.125 \times 172.80 + 0.375 \times 129.60 + 0.375 \times 97.20 + 0.125 \times 72.90 \]

\[ = 115.7625 \]

This is the same outcome as the compounded expected return of 5%, namely

\[ $115.7625 = $100 \times (1.05)^3. \]

As an example, the dashed line through the array represents one possible sequence of outcomes. If we arrive at period T=3 having made the sequence of returns of +20%, +20% and -10% then the arithmetic average return is 10% per period. This is a biased statement of our historical performance for which we have made an average of only 9.03% per period (a total of 29.60% over three periods has a CAGR of 9.03% per period). Compounding 10% for three periods gives an outcome of $133.10 which is biased up compared to the actual outcome of $129.60. Note that the CAGR only depends on the first and last values and not on the journey between them. It does not recognise the risk along the way. The only risk recognised is any implicit risk in the return represented by the ending value.

At period T=3, the dashed line represents our history and it should be analysed using CAGR or continuous growth rates. However, we are seeking an unbiased estimator of the expected return, not a description of the historic return per period. But we can see from the calculations at the bottom of this figure that the arithmetic average gives us an unbiased estimator of our future expected outcomes after compounding up the arithmetic average return. If we repeat the experiment over many times (or use a long history of data assuming each year is a draw from a common random distribution) then we get the following summary table:

**Table 2: Comparison of Averages**

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Arithmetic average</th>
<th>CAGR per period</th>
<th>Average of continuous rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.125</td>
<td>20%</td>
<td>20%</td>
<td>18.23%</td>
</tr>
<tr>
<td>0.375</td>
<td>10%</td>
<td>9.03%</td>
<td>8.64%</td>
</tr>
<tr>
<td>0.375</td>
<td>0%</td>
<td>-0.94%</td>
<td>-0.95%</td>
</tr>
<tr>
<td>0.125</td>
<td>-10%</td>
<td>-10%</td>
<td>-10.54%</td>
</tr>
<tr>
<td>average</td>
<td>5%</td>
<td>4.28%</td>
<td>3.85%</td>
</tr>
</tbody>
</table>
Both the CAGR and the continuous growth rate underestimate the expected outcome. Note that the CAGR is falling. It is converging to the continuous rate. (If your bank started calculating credit card rates as hourly rates instead of daily rates as they do now, then their interest charge on your credit card would converge to the log or continuous equivalent of the discrete rate). Except for pathological cases, the continuous rate average is always less than the discrete average rate because of the relationship

\[ R^C \approx R^D - \frac{1}{2} \sigma^2 \]

where \( \sigma \) is the standard deviation of the distribution of the returns. For a one period return, it is trivial that the arithmetic return is the always the same as the geometric return. The geometric average return (CAGR) starts off equal to the arithmetic return but progressively falls towards the continuous rate.

In summary, the arithmetic average is the appropriate one to use for unbiased forward estimates of expected returns but the CAGR or the continuous rates are the ones to use for historical performance data.